
Short-Long Arm Suspension System Non-Linearities and Analysis

Steven M. Sincere
Specialty Motor Sports

Reprinted From: 1998 Motorsports Engineering Conference Proceedings
Volume 1: Vehicle Design and Safety
(P-340/1)

The appearance of this ISSN code at the bottom of this page indicates SAE's consent that copies of the paper may be made for personal or internal use of specific clients. This consent is given on the condition, however, that the copier pay a \$7.00 per article copy fee through the Copyright Clearance Center, Inc. Operations Center, 222 Rosewood Drive, Danvers, MA 01923 for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law. This consent does not extend to other kinds of copying such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

SAE routinely stocks printed papers for a period of three years following date of publication. Direct your orders to SAE Customer Sales and Satisfaction Department.

Quantity reprint rates can be obtained from the Customer Sales and Satisfaction Department.

To request permission to reprint a technical paper or permission to use copyrighted SAE publications in other works, contact the SAE Publications Group.



GLOBAL MOBILITY DATABASE

All SAE papers, standards, and selected books are abstracted and indexed in the Global Mobility Database

No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

ISSN 0148-7191

Copyright 1998 Society of Automotive Engineers, Inc.

Positions and opinions advanced in this paper are those of the author(s) and not necessarily those of SAE. The author is solely responsible for the content of the paper. A process is available by which discussions will be printed with the paper if it is published in SAE Transactions. For permission to publish this paper in full or in part, contact the SAE Publications Group.

Persons wishing to submit papers to be considered for presentation or publication through SAE should send the manuscript or a 300 word abstract of a proposed manuscript to: Secretary, Engineering Meetings Board, SAE.

Printed in USA

Short-Long Arm Suspension System Non-Linearities and Analysis

Steven M. Sincere
Specialty Motor Sports

Copyright © 1998 Society of Automotive Engineers, Inc.

ABSTRACT

The redistribution of weight due to lateral loading through a short-long arm suspension system requires non-linear methods to accurately predict the steady state deflections throughout the entire vehicle system. An iterative analysis approach which recalculates suspension deflections and movement in the location of center of gravity of the vehicle provides an improvement over linear analysis techniques. Such analysis provides a better understanding of short-long arm suspension systems leading to improved handling at high lateral loading and/or improved noise, vibration and harshness characteristics.

INTRODUCTION

The reactions of automobile suspension systems to lateral loading are inherently non-linear. Spring rates seen at the wheels (wheel rate) vary as a function of the suspension linkage positions, reaction loads into the chassis through the suspension will vary with actuation and the center of gravity of the vehicle will move as the vehicle rolls, pitches and yaws. A linear analysis approach to suspension reactions will not capture these non-linearities and thus give erroneous results for all but the stiffest systems having only minute actuation.

The short-long arm (SLA) suspension system is often chosen in both production and performance automotive applications as the preferred configuration due to the ability of the designer to balance instantaneous roll centers and camber angle changes while providing wheel compliance.

In its basic form, the SLA system is a four bar linkage with a spring most frequently attached to the lower, longer arm and the chassis. The effects of this spring as a flexible link within the system is often overlooked. In a steady state corner, its function is not only to provide wheel compliance, but also to limit body roll in conjunction with, or in the absence of, a stabilizer bar. Approaching that limit, the suspension approaches a "locked" linkage, often resulting in chassis lift, or jacking.

This spring not only compresses and expands with the movement of the lower arm, but rotates about its pinned ends. This change in angular relationship to the lower arm, with its reaction forces from the vehicle weight and cornering forces, will result in changes of the effective load in the direction of the spring, as shown in Figure 1. When viewed from the standpoint of the wheel rate, the spring's effectiveness is reduced as its angle relative to the lower arm deviates from 90 degrees, that is to say that the wheel rate is therefore reduced. This also affects at what point the roll limit is reached.

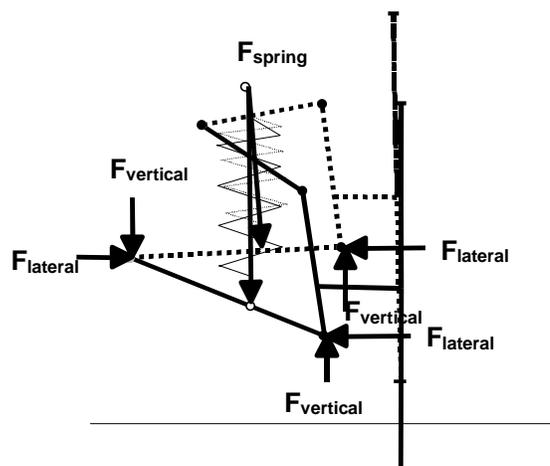


Figure 1. Spring reaction loads.

Perhaps not as obvious is the effect of upper arm position on spring loading. Given that the reaction forces at the lower arm can be derived from the static analysis of the forces acting on the wheel/spindle assembly, Figure 2 illustrates that the vertical reaction at the lower arm is increased or decreased as the upper arm angle is varied from zero. This additionally affects the load seen at the spring, thus varying the wheel rate. Of course, these are the reactions occurring due to instantaneous roll centers and form the basis for classical roll and jacking determination.

Throughout all of this variable loading and wheel rate, the actuation of the suspension results in a rolling and jacking of the chassis, and to a lesser extent a pitching and yawing. As the center of gravity of the chassis moves, so changes the reaction loads at the tire contact patches according to free body diagram (FBD) analysis. Thus, any initial calculation of weight transfer from inside to outside wheels becomes obsolete and must be adjusted to determine accurate wheel loading.

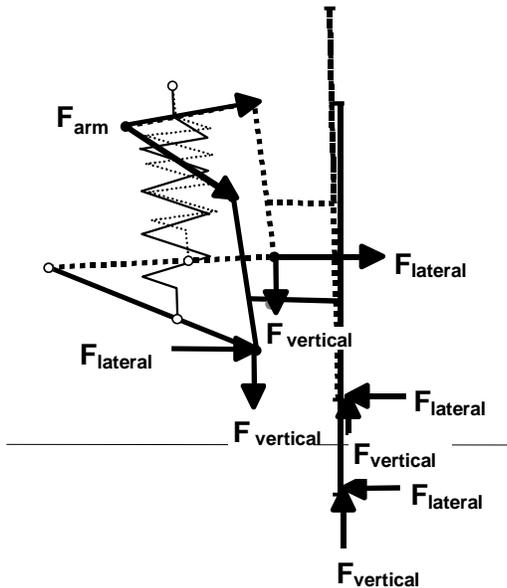


Figure 2. Upper arm reaction loads.

Through an understanding of these effects and their interactions, SLA systems can be improved in order to achieve such things as elimination of stabilizer bars through balancing of front and rear systems or spring rate selection, improved handling through softer spring rates and camber change optimization, all which lead to reduction of associated chassis flex and noise, vibration and harshness.

PROPOSED ANALYTICAL METHOD

The purpose of this work was to develop an analytical technique in order to study these non-linear effects. The technique is based on a simplified, two wheel (one end of a vehicle) system. The motion of the linkages are planar (within the vertical and lateral plane). The analysis is done assuming a steady state turn with no banking, no surface irregularities and no aerodynamic effects. Tire performance characteristics are assumed to be linear (both the inside and outside tire are subjected to a lateral load proportional to their vertical load) and no sliding occurs. Stiffness of the springs are linear and the chassis, bushings, arms, tires and wheels are assumed infinitely stiff.

The method begins with the vehicle at rest with only the force of gravity acting upon it. It is assumed that the vehicle is at steady state ride height, thus the springs are

bearing the appropriate load to support the chassis. Static analysis is performed throughout the suspension system in order to calculate these initial spring loads.

The reactions in a steady state turn are then determined with the help of a free body diagram of the vehicle as affected by gravity and the centrifugal forces encountered, as shown in Figure 3. The vertical reaction loads on the inside and outside tires are calculated through simple static analysis. As stated in the assumptions above, each tire is assumed to bear a percentage of the total lateral load in proportion to their vertical load. (Although not included in this study, further enhancements are possible through including iterative steps at this point to determine effects of actual tire performance curves on lateral load bearing of each wheel.)

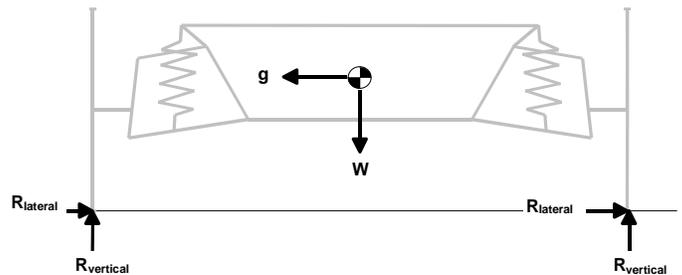
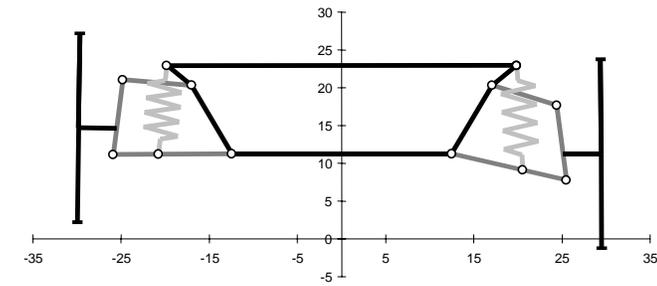


Figure 3. Overall vehicle free-body diagram.

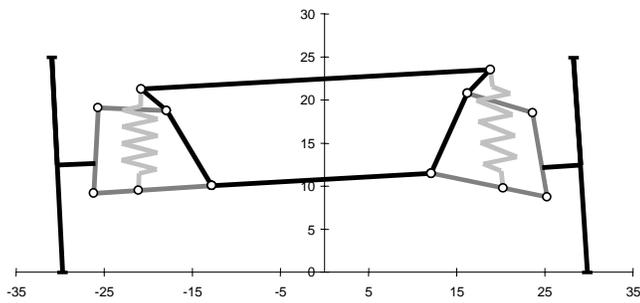
These vertical and lateral tire loads are then analyzed through the suspension system to the springs. As the spring loads within the turn are determined, the difference from the resting spring loads is calculated. This difference, coupled with the stiffness of the springs is used to calculate a length change of the spring. The effects of this length change is then applied to the geometry throughout the suspension system to arrive at wheel deflections relative to the chassis, as shown in Figure 4a. Once this “deformed geometry” is determined, the entire vehicle (chassis, center of gravity, and suspension geometry) is rotated and translated back to the original horizontal axis (Figure 4b). This new geometry then provides the starting point for the next iteration, beginning with the free body diagram.

Iterations through this process are continued until an acceptable convergence is achieved. In the case study to follow, spring load was selected as the variable measured for convergence. Other variables such as chassis or wheel deflection can be used, and were also reviewed relative to iteration cycle.

As a side note, certain combinations of vehicle mass, suspension geometry and spring rates can make convergence difficult to achieve. Although this was not seen in the case study, this is overcome through successive load steps through various lateral g-loads, with each step beginning at the resulting converged geometry of the preceding.



(a) Fixed chassis suspension deflection.



(b) Suspension + chassis deflection.

Figure 4.

CASE STUDY

Figure 5 depicts the configuration studied using the aforementioned method. It included a sprung vehicle mass of 1000 lbs (453.6 kg) being supported by the wheel pair located at 16 inches (40.64 cm) above the road surface. The spring rate selected for the analysis was 500 lbs/in (875.6 N/cm).

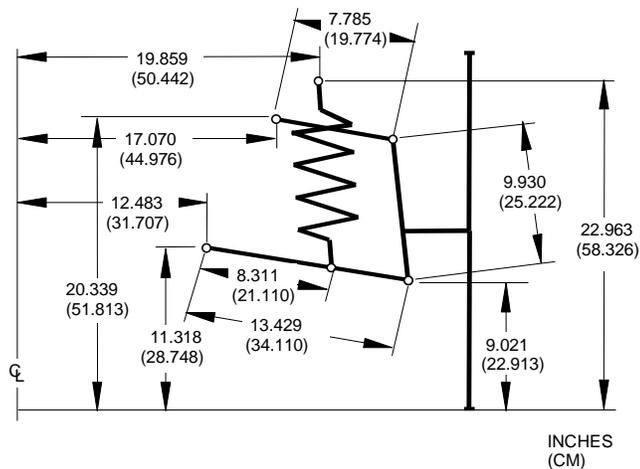


Figure 5. Case study geometry.

Kinematic analysis was performed on this system to determine camber change, lateral tire movement (scrub), instantaneous roll centers and effective wheel rates over a range of wheel travel +3.082 inches (7.828 cm) and -2.832 inches (-7.193 cm) from ride height position relative to the chassis.

As seen in Figure 6, camber changes with wheel deflection exhibit typical characteristics of an SLA system. Negative camber is achieved on compression and increases throughout the travel. This is the desired effect in order to maintain tire perpendicularity to the road surface on the outside tire (therefore best contact patch) as chassis roll is experienced within a turn. However, negative camber is also achieved on the expansion of the system, causing poor contact patch geometry on the inside tire. This is often dismissed through assumption that the outside tire carries most or all of the lateral loading, especially in high performance, high lateral loading situations.

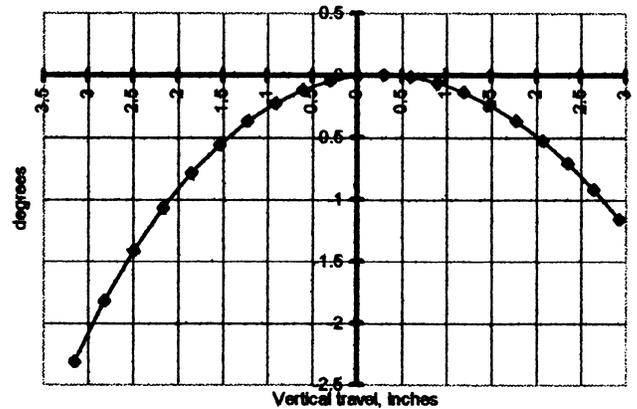


Figure 6. Camber vs. wheel vertical travel.

Tire scrub during suspension actuation can have the effect of binding the system and should be kept to a minimum. The scrub characteristics of this system, as shown in Figure 7, are such that there is .360 inches (.914 cm) of scrub at maximum compression and .530 inches (1.346 cm) at maximum expansion. These values fall within 10% of a typical tire section height, and therefore sidewall flexibility should absorb this under all circumstances.

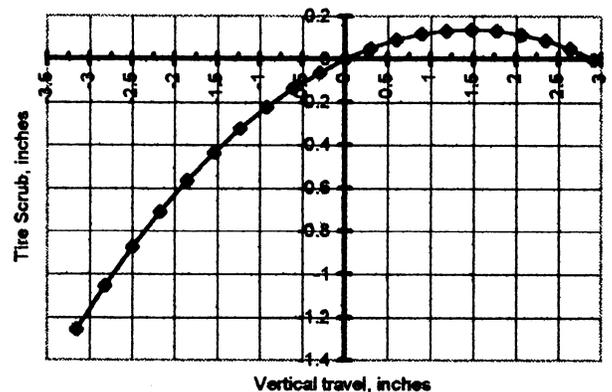


Figure 7. Tire scrub vs. wheel vertical travel.

Figure 8 shows the instantaneous roll center of the vehicle relative to the chassis and the road (for reference) as both wheels are compressed or expanded together. Note that relative to the chassis, the instantaneous roll center rises slightly on compression (thus reducing the tendency to roll) and drops abruptly in expansion (tending to increase roll). Of course, each wheel will be moving inde-

pendently in a turn and the instantaneous roll center will move both vertically and laterally because of this. These instantaneous roll center values are worth noting when comparing the results of this study to classical roll and jacking analysis.

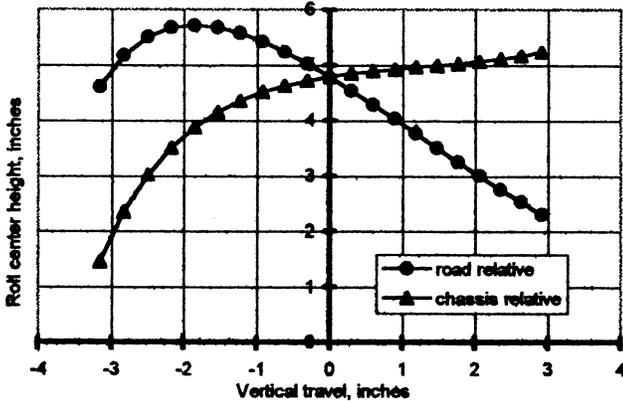


Figure 8. Roll center vs. wheel vertical travel.

All of the above results were by-products of the investigation of the non-linearity of the wheel rates relative to kinematic motion. Figure 9 describes this for the spring mounts in the baseline position. Note that the wheel rate is nearly flat on compression with only a slight drop from ride height position to 1.2 inches of compression yet increase of 8.4% to maximum expansion. Due to this effect alone, one would expect the outside suspension to compress more than the inside suspension expands (given equal weight transfer) within a turn, thus resulting in a lowering of the chassis rather than jacking.

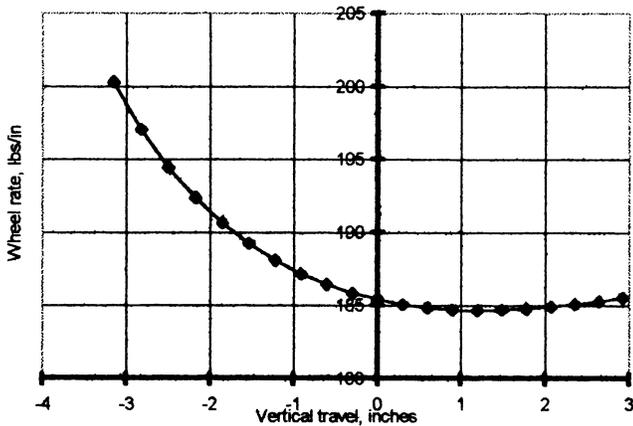


Figure 9. Wheel rate vs. wheel vertical travel.

Using the methods previously described, this geometry was analyzed to determine deflection reaction to lateral loading from 1/8 to 1 g. The reactions relative to camber change of the tires and roll and jacking of the chassis were calculated.

As shown in figure 10, this geometry produces an increasing positive camber change on the outside tire throughout the lateral loading range. Meanwhile, the inside tire is experiencing the opposite effect with an increasing negative chamber change. It becomes obvious that at the extreme handling conditions (approaching

1g) the tire contact patch will be distorted by these camber changes and thus reduce effective cornering capability.

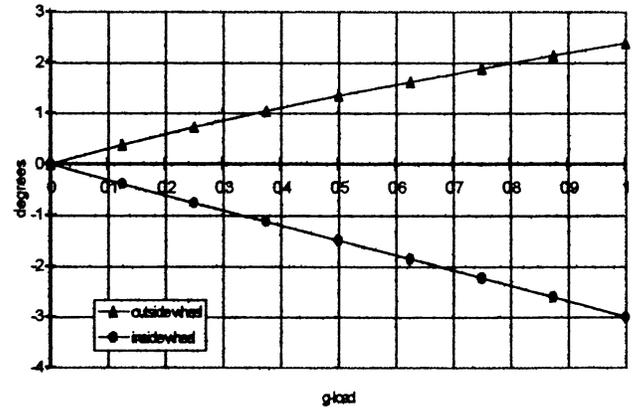


Figure 10. Camber change vs. lateral g-load.

This adverse change in tire cambers is explained by review of the chassis roll results. Figure 11, when compared to figure 6, shows that the chassis roll far exceeds the maximum achievable tire camber changes available throughout the kinematics of the SLA system. One apparent culprit is limited compression on the outside suspension system. Load transfer to the outside wheel resulted in an increase of 505 pounds (2248 N). Working backward through figure 9 results in a vertical wheel travel of 1.697 inches (4.310 cm). Referring to figure 6 shows that this compression only produces a chassis relative camber change of .33°.

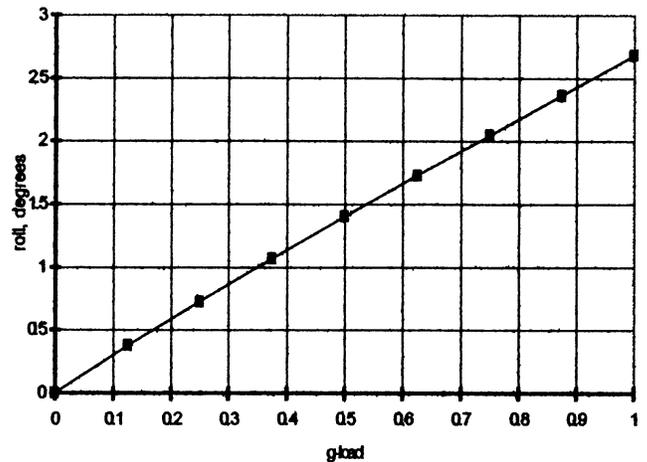


Figure 11. Chassis roll vs. lateral g-load.

The anticipated result is that such roll and limited outside system deflection (coupled with a relatively high instantaneous roll center) would result in considerable chassis jacking, yet that is not the case as is shown in figure 12. This is explained through review of the wheel rates in figure 9. The reduction in wheel rate upon compression (and increase upon expansion) leads to larger wheel height changes on the outside tire. The net result is a sort of dive into the turn as the outside suspension compresses more than the inside suspension expands. Thus, these results produce some challenge to the classical

conceptions of instantaneous roll centers and jacking effects.

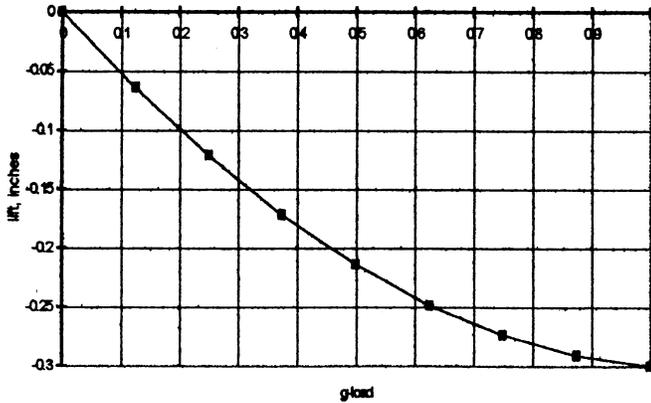


Figure 12. Chassis jacking vs. lateral g-load.

Up to this point, only the results of the analysis have been discussed. The importance of this work is in understanding the error associated with the assumption of linear reactions. The nonlinear effects are greatest at the highest lateral loading, therefore, the convergence of the analysis method was most apparent at the 1g case in the study. As shown in Figure 13, calculated spring load error is less than 1% after only 4 iterations for this condition. However, note that the first iteration (which represents linear behavior) shows considerable error with the loading in the outside spring 12.75% higher than the converged (tenth iteration) solution and the inside spring loading 5.49% lower. This amount of error at the springs translates to error in chassis roll of .62° and camber changes of +.44° and -.66° for the outside and inside wheels respectively. Errors of this magnitude in high performance applications are significant and warrant the use of an iterative, non-linear analysis as presented here. Figure 14 shows the error in spring loading due to linear assumption over the entire lateral loading range of the study.

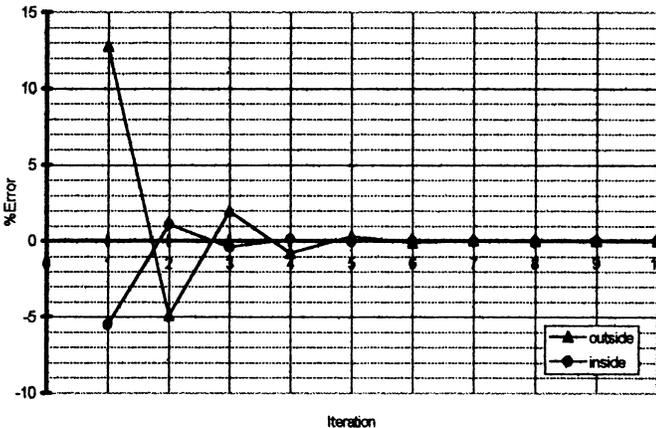


Figure 13. Spring load error relative to 10th iteration at 1g lateral load.

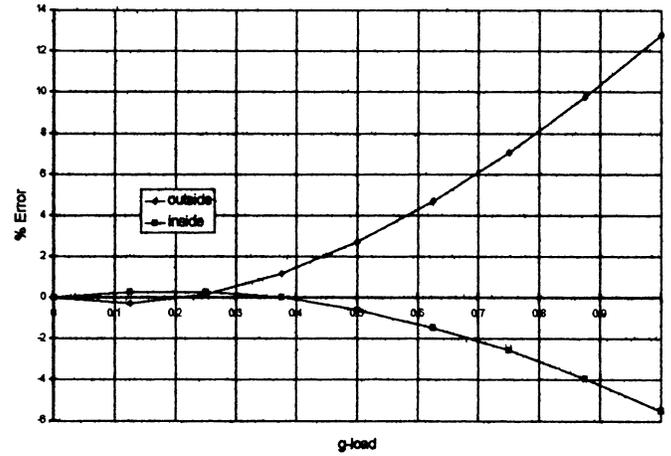


Figure 14. Linear analysis error.

As was previously recognized, wheel rates are non-linear in part due to the variation of spring angle relative to the suspension linkages. In an effort to determine a method of adjusting the wheel rate curve with minimal linkage configuration changes, the chassis mount point of the spring was varied and the spring rate was held constant. From the baseline position, the spring angle was moved $\pm 13.5^\circ$. (13.5° was the angle change required to provide an initial spring position perpendicular to the lower control arm, and was thus also chosen to be the same angle change in the opposite direction to complete the range of the study.) These spring positions are shown in figure 15. The effect on wheel rates is shown in figure 16, illustrating an increase in initial wheel rate as the spring approaches perpendicularity to the lower arm and reduced initial wheel rate as this angle decreases, as would be expected. The point of interest is the change in relative shape of the wheel rate curve. The perpendicular (or outward tilted) spring results in a wheel rate that decreases on compression and increases on expansion more rapidly than that of the baseline. Conversely, the inward tilted spring produces a progressive wheel rate on compression and a more constant wheel rate on expansion.

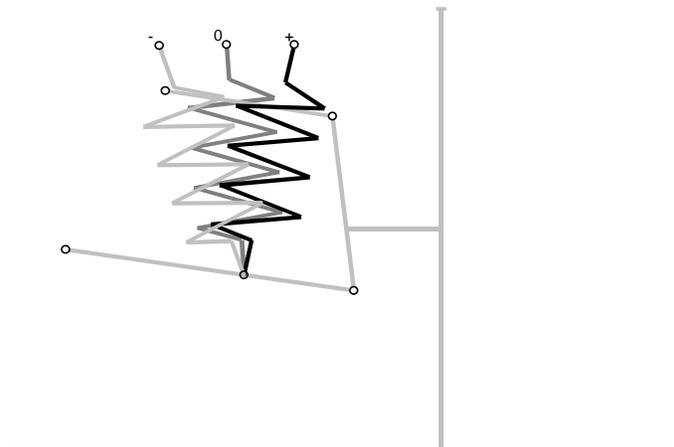


Figure 15. Spring upper pivots.

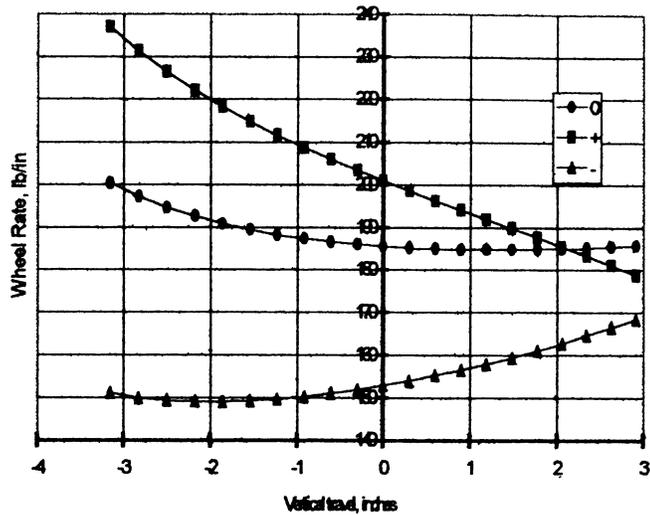


Figure 16. Wheel rates at various spring angles vs. wheel vertical travel.

The effect these wheel rates have on the chassis roll and tire camber characteristics is given in figures 17 through 19. With the springs tilted outward, there are nearly no changes in roll and wheel camber responses to lateral loading. However, there is an increase in negative jacking, or diving into the turn, which is easily explained above by the wheel rate curve. Thus, this modification only serves to increase the wheel rate and negative jacking, leading to a rougher ride, decrease in road following capability of the tire and less ground clearance in turns...all undesirable effects. Meanwhile, the tilted inward spring position results in more chassis roll (due to lower wheel rates), slightly more adverse camber changes (due to increased roll) and slightly more negative jacking at lower lateral loading (less than .6g). At this point it is apparent that spring mounting points provide some adjustability in the handling characteristics of the vehicle and that a variety of spring rates and angles can be analyzed in order to optimize a given SLA system.

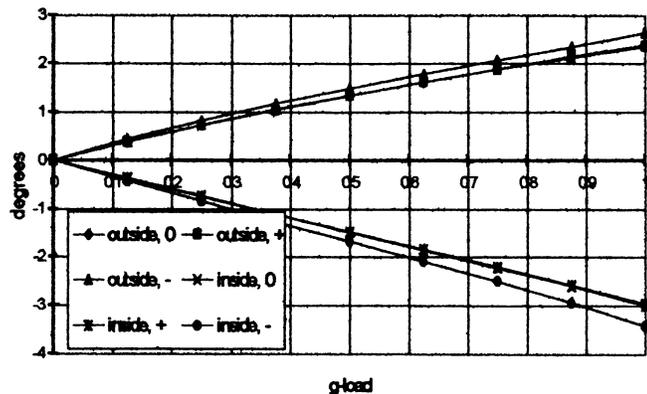


Figure 17. Camber change vs. lateral g-load.

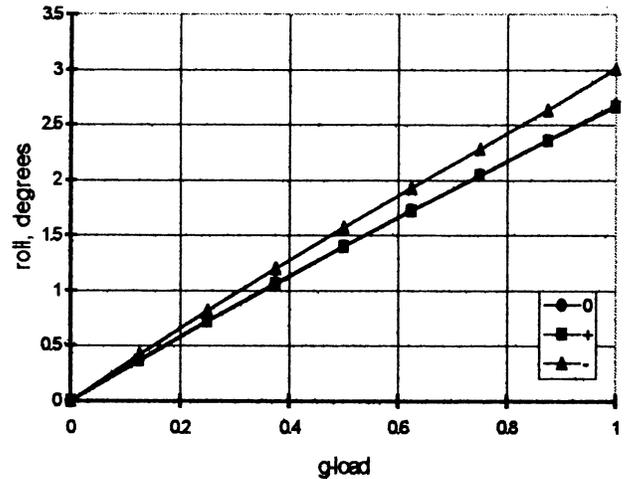


Figure 18. Chassis roll vs. lateral g-load.

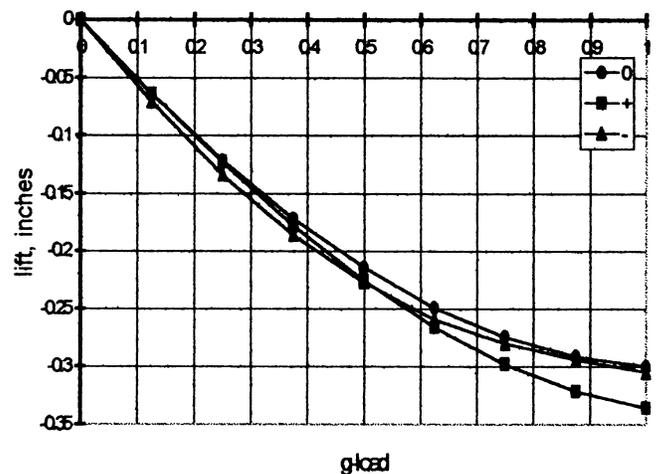


Figure 19. Chassis jacking vs. lateral g-load.

CONCLUSION

The preceding case study illustrated that the proposed iterative analysis method provides a better understanding of the non-linear reactions of the SLA suspension system while minimizing potential error associated with linear analysis. Use of this method, or other nonlinear techniques, provides the ability to:

1. Optimize suspension geometry for maintaining desired camber angles and minimum spring/wheel rates,
2. Study variable spring rates inherent to a particular SLA system or progressive spring rate mechanisms,
3. Determine optimal weight and spring bias for performance applications in oval track racing,
4. Match the characteristics of the front system to the rear, or vice versa.

And, finally, this work forms the basis for development of methods for other types of suspension systems and a four wheel system analysis which includes pitch and yaw characteristics.