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Formula SAE Suspension Design

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ABSTRACT

Resume a Formula SAE suspension design. After rules analysis, which limits the suspension a minimum travel and wheelbase, project targets were defined, than a benchmarking was made on top teams. The tire behavior is discussed. The unequal A-arms with tie-rod on front and rear suspension are detailed. The dimensional approach was developed on CAD concerning dimensional restrictions. The transient stability, control and maneuvering performance were analyzed on overall forces and moments diagram. For kinematics and dynamic analysis a multibody model was used. Rollover, ride and handling were simulated and were done tuning on geometry, springs and dampers to achieve performance.

INTRODUCTION

The student competition Formula SAE (FSAE), sponsored by the Society of Automotive Engineers (SAE), motivate students to design, build and compete with a small formula style race car. The basis of the competition is that a fictitious company has contracted a group of engineers to build a small formula car. The first step are the competition rules analysis, which limits the suspension system with a minimum wheel travel of 50mm, a wheelbase greater than 1524mm. FSAE suspensions operate in a narrow realm of vehicle dynamics mainly due to the limited cornering speeds which are governed by the racetrack size, with 140km/h as top speed and 60km/h as turn top speed. The dynamic portions of the competition are the 15.25 m diameter skid-pad, 91.44 m acceleration event, 0.8 km autocross, 44 km endurance race.

The project targets were defined. Than a benchmarking between the top ten teams was made. The suspension geometry section concentrates on some of the basic areas of suspension design and highlights. Therefore, FSAE suspension design should focus on the constraints of the competition. For example, vehicle track width and wheelbase are factors governing the success of the car's handling characteristics. These two dimensions not only influence weight transfer, but they also affect the turning radius. The targets were at fist accept the rules, than low system weight, create maximum mechanical grip, provide quick response, transmit accurate driver feedback and be able to adjust balance.

TIRE AND WHEEL

The suspension design process used an "outside-in" approach by selecting tires that meet the vehicle requirements, and then designing the suspension to suit the tire parameters. Short race durations, low vehicle mass, and low-speed courses all indicate a need for a tire that reaches its operating temperature quickly. The tire is important to the handling of the vehicle, the design team should thoroughly investigate the tire sizes and compounds available. The tire size is important at this stage of the design since the height of the tire must be known before the suspension geometry can be determined. For example, the tire height for a given wheel diameter determines how close the lower ball joint can be to the ground if packaged inside the wheel.

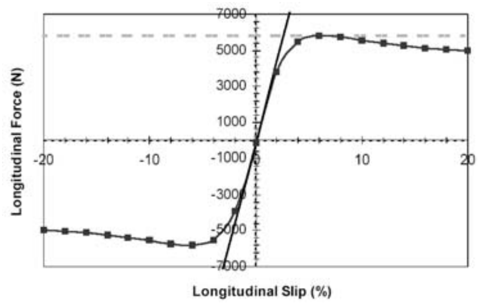

Figure 1- Tire longitudinal force.

The designers should be aware that the number of tire sizes offered for a given wheel diameter is limited.

Therefore, considering the importance of the tire to handling, the tire selection process should be methodical. Since the amount of tire on the ground has a large influence on grip, it is sometimes desirable to use wide tires for increased traction. However, it is important to remember that wide tires add rotating mass, which must be accelerated by a restricted FSAE engine.

This added mass might be more detrimental to the overall performance than the increase in traction from the wider tires. Not only does a wider tire add mass, but it also increases the amount of rubber that must be heated. Since racing tires are designed to operate most efficiently in a specific temperature range, this added material may prevent the tires from reaching the optimum temperature range.

During the selection process the designers must consider how the tires will influence the performance of the entire package. For example, the weather conditions for the FSAE dynamic events might determine which tire compound and tire size should be used for the competition. This tire selection increased the operating temperature from 48°C to 60°C. The team chose to use the harder compound since the weather for the endurance was predicted to be clear and warm.

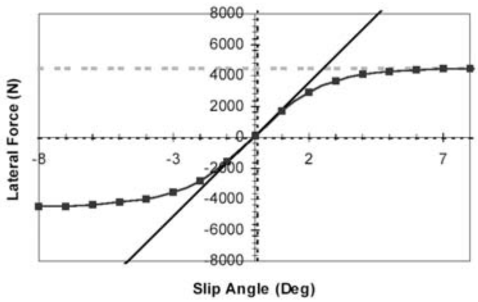

Figure 2- Tire lateral force.

Based on these, the Hoosier 20"x6.0", R25A compound was selected. Infrared tire temperature and two-axis acceleration data logging showed it best achieved the tire goals. It was also exceptionally compliant, was the lightest road-race tire tested, and had the lowest mass moment of inertia.

Once a decision has been made as to which tire sizes to use, the wheel selection should be next. Usually, the wheel dimensions are fixed and allow for little modification. Therefore, it is important to have some design goals in mind before investing in wheels. Generally, the upright, brake caliper, and rotor are placed inside the wheel, which requires wheel offset for clearance. It is usually easier to design the suspension geometry if the wheel profile is known. For example, the ball joint location is limited to the area defined by the wheel profile.

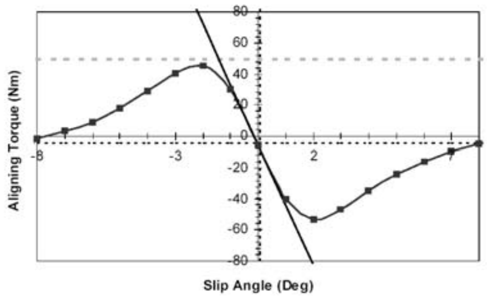

Figure 3- Tire aligning torque.

Other considerations for wheel selection include: cost, availability, bolt circle, and weight. For example, three-piece rims, although expensive, have the distinct advantage of offering many offsets and profiles that can be changed during the design process. All four wheels selected were size 6 by 13. This wheel selection allowed for tire rotations, reduced cost, and wide selections of tire sizes, compounds, and manufacturers.

CONCEPT AND DIMENSIONAL APPROACH

As much in the front suspension as in the rear was adopted double A-arm and toe link, with pushrod. Its adjustment aims at one better optimization to each event of the competition. In the beginning of the project, measures as wheelbase, cg position, wheel and tire dimensions, had been adopted on the basis of benchmark and evaluated in a bicycle model (*Simulink*).

Track width is the distance between the right and left wheel centerlines. This dimension is important for cornering since it resists the overturning moment due to the inertia force at the center of gravity (CG) and the lateral force at the tires. For the designer, track width is important since it is one component that affects the amount of lateral weight transfer. Also, the designers must know the track width before kinematics analysis of the suspension geometry can begin. When selecting the track width, the front and rear track widths do not necessarily have to be the same. For example, track width is typically wider in the front for a rear wheel drive race car. This design concept is used to increase rear traction during corner exit by reducing the amount of body roll resisted by the rear tires relative to the front tires [2]. Based on the corner speeds and horsepower-to-weight ratio of FSAE cars, this concept should be considered by the designer.

The wheelbase also needs to be determined. Wheelbase is defined as the distance between the front and rear axle centerlines. It also influences weight transfer, but in the longitudinal direction. Except for anti-dive and anti-squat characteristics, the wheelbase relative to the CG location does not have a large effect on the kinematics of the suspension system.

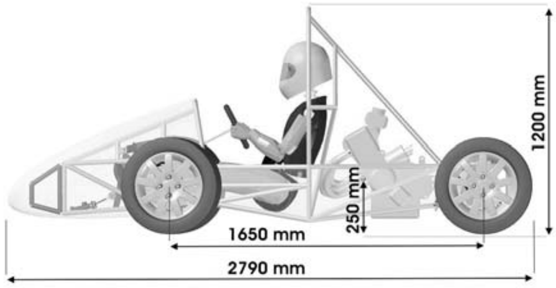

Figure 4- Formula lateral view.

However, the wheelbase should be determined early in the design process since the wheelbase has a large influence on the packaging of components. A shorter wheelbase provides faster system response due in part to reduced yaw moment of inertia and transient tire effects. This quick response is important on tight circuits. For track width and wheelbase starting points, the designers should research the dimensions of the opposition cars to serve as a baseline for their own calculations. FSAE car specifications for the competing teams, including track width and wheelbase, are available in the event program published by SAE.

KINEMATICS

The designer can now set some desired parameters for the suspension system. These usually include camber gain, roll center placement, and scrub radius. The choice of these parameters should be based on how the vehicle is expected to perform. By visualizing the attitude of the car in a corner, the suspension can be designed to keep as much tire on the ground as possible. For example, the body roll and suspension travel on the skid pad determines, to a certain extent, how much camber gain is required for optimum cornering. The amount of chassis roll can be determined from roll stiffness while the amount of suspension travel is a function of weight transfer and wheel rates.

Once a decision has been made about these basic parameters, the suspension must be modeled to obtain the desired effects. Before the modeling can begin, the ball joint locations, inner control arm pivot points, and track width must be known. The easiest way to model the geometry is with a kinematics computer program since the point locations can be easily modified for immediate inspection of their influence on the geometry. Should a dedicated kinematics computer program not be available, and then simply redrawing the suspension as the points are moved can use a CAD program. When designing the geometry, it is important to keep in mind that designing is an iterative process and that compromises will be inevitable.

At the CAD model was analyzed parameters as roll center, suspension sizes arms, scrub radius, anti-squat, anti-dive and caster, camber and kingpin angles. Choused the first configuration, it became optimization in MBS (*ADAMS/Car*). Wheels parallel displacement, opposing displacement, chassis rolling; steering and static loads simulation had been carried.

For instance, the desired scrub radius might not be possible because of packaging constraints. When modeling the suspension, the designers should not aimlessly modify points without first thinking through the results. For example, the designer should visualize how the wheel would camber relative to the chassis when making the lower A-arm four times longer than the upper A-arm. One method that can be used to visualize the results is the instant center location for the wheel relative to the chassis. Another method is to use the arcs that the ball joints circumscribe relative to the chassis. For a complete explanation about determining suspension point locations from instant center locations refer to Milliken [2]. Scrub Radius, Kingpin Inclination, and Caster.

The scrub radius, or kingpin offset, is the distance between the centerline of the wheel and the intersection of the line defined by the ball joints, or the steering axis, with the ground plane. Scrub radius is considered positive when the steering axis intersects the ground to the inside of the wheel centerline. The amount of scrub radius should be kept small since it can cause excessive steering forces. However, some positive scrub radius is desirable since it will provide feedback through the steering wheel for the driver.

Kingpin inclination (KPI) is viewed from the front of the vehicle and is the angle between the steering axis and the wheel centerline [1]. To reduce scrub radius, KPI can be incorporated into the suspension design if packaging of the ball joints near the centerline of the wheel is not feasible. Scrub radius can be reduced with KPI by designing the steering axis so that it will intersect the ground plane closer to the wheel centerline. The drawback of excessive KPI, however, is that the outside wheel, when turned, cambers positively thereby pulling part of the tire off of the ground. However, static camber or positive caster can be used to counteract the positive camber gain associated with KPI.

Caster is the angle of the steering axis when viewed from the side of the car and is considered positive when the steering axis is tilted towards the rear of the vehicle [9]. With positive caster, the outside wheel in a corner will camber negatively thereby helping to offset the positive camber associated with KPI and body roll. Caster also causes the wheels to rise or fall as the wheel rotates about the steering axis, which transfers weight diagonally across the chassis. Caster angle is also beneficial since it will provide feedback to the driver about cornering forces. The suspension design team chose a scrub radius of 9.5mm, 7 degrees of KPI, and 4 degrees of caster. This design required the ball joints to be placed near the centerline of the wheel, which required numerous clearance checks in the solid modeling program.

The a-arm mounting point geometry must be defined. The points of the lower a-arm are generated largely by packaging, as the outer point must be as low as the wheel will allow, and be as wide as the track width and frame will allow. The steer axis must be defined to place the upper, outer point. Analyses of the effects of steer angle, caster, KPI, and scrub radius show that a value of 4° of caster provides roughly optimal front camber for a typical steer angle. KPI will be minimized in packaging because of its adverse camber effects, and scrub radius will be increased to improve driver feedback in one-wheel lockup situations and to reduce understeer moments in tight hairpins.

Once the basic parameters have been determined, the kinematics of the system can be resolved. Kinematics analysis includes instant center analysis for both sets of wheels relative to the chassis and also for the chassis relative to the ground. The points labeled IC are the instant centers for the wheels relative to the chassis. The roll center is the point that the chassis pivots about relative to the ground. The front and rear roll centers define an axis that the chassis will pivot around during cornering. Since the CG is above the roll axis for most race cars, the inertia force associated with cornering creates a torque about the roll center. This torque causes the chassis to roll towards the outside of the corner. Ideally, the amount of chassis roll would be small so that the springs and anti-roll bars used could be a lower stiffness for added tire compliance. However, for a small overturning moment, the CG must be close to the roll axis. This placement would indicate that the roll center would have to be relatively high to be near the CG. Unfortunately, if the roll center is anywhere above or below the ground plane, a "jacking" force will be applied to the chassis during cornering. For example, if the roll center is above ground, this "jacking" force causes the suspension to drop relative to the chassis.

Suspension droop is usually undesirable since, depending on the suspension design, it can cause positive camber, which can reduce the amount of tire on the ground. Conversely, if the roll center is below the ground plane, the suspension goes into bump, or rises relative to the chassis, when lateral forces are applied to the tires. Therefore, it is more desirable to have the roll center close to the ground plane to reduce the amount of chassis vertical movement due to lateral forces. Since the roll center is an instant center, it is important to remember that the roll center will move with suspension travel. Therefore, the design team must check the migration of the roll center to ensure that the "jacking" forces and overturning moments follow a relatively linear path for predictable handling. For example, if the roll center crosses the ground plane for any reason during cornering, then the wheels will raise or drop relative to the chassis, which might cause inconsistent handling. The roll center is 35.6mm below ground in the front and 35.6mm above ground in the rear.

Because of the large roll moment, the team designed enough camber gain into the suspension to compensate for body roll associated with soft springs and no anti-roll bar. Camber is the angle of the wheel plane from the vertical and is considered to be a negative angle when the top of the wheel is tilted towards the centerline of the vehicle. Camber is adjusted by tilting the steering axis from the vertical, which is usually done by adjusting the ball joint locations. Because the amount of tire on the ground is affected by camber angle, camber should be easily adjustable so that the suspension can be tuned for maximum cornering. For example, the amount of camber needed for the small skid pad might not be the same for the sweeping corners in the endurance event. The maximum cornering force that the tire can produce will occur at some negative camber angle [7].

However, camber angle can change as the wheel moves through suspension travel and as the wheel turns about the steering axis. Because of this change, the suspension system must be designed to compensate or complement the camber angle change associated with chassis and wheel movements so that maximum cornering forces are produced. The amount of camber compensation or gain for vertical wheel movement is determined by the control arm configuration. Camber gain is usually obtained by having different length upper and lower control arms. Different length control arms will cause the ball joints to move through different arcs relative to the chassis. The angle of the control arms relative to each other also influences the amount of camber gain. Because camber gain is a function of link geometry, the amount of gain does not have to be the same for both droop and bump.

For example, the suspension design might require the wheels to camber one degree per 25mm of 5 droop versus negative two degrees per 25mm of bump. Static camber can be added to compensate for body roll; however, the added camber might be detrimental to other aspects of handling. For example, too much static camber can reduce the amount of tire on the ground, thereby affecting straight line braking and accelerating. Similarly, too much camber gain during suspension travel can cause part of the tire to loose contact with the ground. Caster angle also adds to the overall camber gain when the wheels are turned. For positive caster, the outside wheel in a turn will camber negatively, while the inside wheel cambers positively. The amount of camber gain caused by caster is minimal if the wheels only turn a few degrees. The use of low wheel rates with a large roll moment required the suspension to compensate for the positive camber induced by chassis roll and suspension travel. The camber gain was from both the caster angle and the control arm configuration.

RIDE

The primary function of a suspension system is to isolate the road excitations experienced by the tires from being transmitted to the passengers. At race cars this approach is ignored to improve better performance, so the suspension must guarantee less vertical force variation. Time domain statistics, such as mean suspension deflection, maximum and RMS values of suspension acceleration are often used in suspension design as criteria Multi-body dynamics has been used extensively by automotive industry to model and design vehicle suspensions.

The mathematical model used for simulating vehicle dynamic is known as "quarter car". It is traditionally used for analysis of the ride dynamics of passenger cars. In the quarter-car model, the effects of vehicle roll are assumed to be negligible. The effects of pitch are considered by increasing the sprung mass by 100% from its original value such that the vertical motion caused by vehicle pitch (the mass moment of inertia) is incorporated.

Therefore, the quarter-car model is a two degree-of-freedom system with the vertical displacements. The road profile elevation is the input to the system. The spring-damper system with stiffness and damping coefficient represents the linear model of tire, which has constant point contact with the road.

The experimental road profile data used in the simulation was sampled by UMTRI in 1989-1996 on interstate PA42 with a sampling interval of 0.152meter. As determined by the sampling interval, the maximum wave number presented in the road profiles is 3.3cycles/meter. The profile is 500 meters in length, with an IRI of 170in/mile. This value represents the average road condition defined by FHWA.

The procedure to achieve spring stiffness (17N/mm in the front and rear) was based on complete vehicle frequency analysis (table 1). The applied methodology is described in *Gillespie* [1]. For damping curves determination was used *Milliken* [2].

Table 1-Vehicle frequency.

Nº	[Hz]	Mode description
1	1,05	Bounce - Pitch
2	1,48	Pitch - Bounce
3	4,17	Roll
4	18,19	Vertical rear axle
5	18,56	Vertical front axle
6	18,98	Roll rear axle

Inboard rocker pivot points and damper locations allow rapid changes wheel rates and their linearity. A pushrod-rocker setup also allows easier access to the dampers, and raises the CG only 6mm compared to a pull-rod setup. To provide the maximum mechanical grip, wheel rates were selected to be as compliant as possible without allowing the chassis to contact the ground. These values were compared to successful values found during testing. Modified coil-over bicycle dampers are selected for their low cost, light weight, compactness, and compatibility with springs appropriate for low vehicle weights, in comparison to typical auto racing dampers. A recent dyno test has shown that these dampers are consistent within 4%.

ROLLOVER

The model to predict vehicle propensity to rollover includes the effects of suspension and tire compliance. The model uses only a few parameters, usually known at the design stage. The lateral accelerations at the rollover threshold predicted by the model are compared to the results of simulations, in which vehicles with the same static stability factor, but different suspension characteristics and payloads are subjected to roll inducing handling maneuvers. The results of simulations correlate well with the predictions based on the proposed model. Design recommendations for passive suspensions, which would increase rollover stability are discussed.

Static stability factor is obtained by considering the balance of forces acting on a rigid vehicle in steady state cornering. During cornering the lateral tire forces on the ground level (not shown) counterbalance the lateral inertial force acting at vehicle center of gravity, resulting in a roll moment. This moment is counterbalanced by the moments of vertical forces. At the limit cornering condition (rollover threshold) the normal load reaches zero.

Neglecting the compliances of suspensions and tires leads to overestimation of rollover threshold. During cornering vehicle, body rolls about the roll axis, resulting in the lateral shift of vehicle center of gravity towards outside of turn. At the same time, lateral forces of the outside tires cause lateral deformation of the tires and camber. All these factors contribute to the reduction of the moment arm of the gravity force, which acts to stabilize the vehicle. At the same time, vertical movement of the wheel with respect to the body is usually accompanied by the lateral movement, which can change the half-track width. This lateral movement is determined primarily by suspension kinematics. In addition, the lateral forces are transmitted between the body and the wheels by rigid suspension arms, which in general are not parallel to the ground. Therefore these link forces have vertical components, which in general do not cancel out and may elevate vehicle center of gravity.

The final result is the reduction of the effective half-track width and usually increases in height of vehicle center of gravity, both of which reduce the rollover threshold. In addition, during cornering vehicle wheels rotate about the lateral axis and concurrently about the vertical axis (the axis of vehicle turn). This results in gyroscopic moments, which contribute to the moment equation. Finally, in dynamic maneuvers, the roll angle of vehicle body may exceed (overshoot) the steady-state value. The amount of overshoot depends on the type of maneuver, but for a given maneuver it is related to the roll damping of suspension as well as suspension stiffness and the body moment of inertia about the roll axis. In what follows, each effect is discussed separately and simplified equations are provided which describe their impact on rollover threshold as function of known vehicle parameters.

In steady state cornering there are primarily two sources of jacking forces: nonlinearities in suspension stiffness characteristics and vertical components of forces transmitted by suspension links. Suspension stiffness characteristics are usually progressive, that is stiffness increases with suspension deflection in order to maintain good ride properties with a full load. During cornering maneuvers, progressive characteristic of suspension permits smaller deflection in compression of the outside suspension than deflection in extension of the inside suspension. As a result, height of vehicle center of gravity increases. The second jacking effect is a result of forces in suspension links. Lateral forces generated during cornering maneuvers are transmitted between the body and the wheels through relatively rigid suspension links. In general these members are not parallel to the ground; therefore the reaction forces in these elements have vertical components, which usually do not cancel out, resulting in a vertical net force, which pushes the body up.

HANDLING

A vehicle system dynamics model is presented that captures the essential braking and handling behavior of an automobile with independent suspensions on a flat surface. It is often said that an automobile is controlled by forces developed in just four small patches, each the size of a man's hand, where the tires contact the road. The new computer models were more complex, typically with 10 to 20 DOF [3, 4]. The additional complexity accounted for nonlinearity and more detailed suspension kinematics. Starting with the mid-1980's, engineers started using newly available multibody simulation programs to describe the model geometrically, "assembling" the system model from components [5]. Modelers no longer had to derive equations, and therefore, the efforts and potential errors associated with deriving equations and coding them were nearly eliminated.

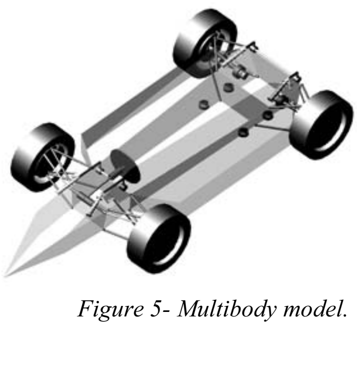

Figure 5- Multibody model.

Automotive manufacturers and many others now use multibody programs to perform simulations of automotive handling and braking behavior [7]. The tendency has been to include nearly all-moving parts in the suspensions and steering systems. Inputs include coordinates of most joints between parts, and mass properties of individual parts. The advantage of the detailed multibody programs for development engineers is that they can fine-tune designs by modifying component-level details.

The major ingredients for describing the rigid-body kinematics and dynamics are bodies, points, and vector directions. Forces will be defined in terms of magnitudes, directions, and points that lie on the lines of action. The magnitudes and directions will usually be described in terms of quantities such as position and velocity vectors that are available from the multibody program. Some of the model degrees of freedom (DOF) are handled with auxiliary user-defined state variables and equations.

There are just three governing equations: the sum of the tire shear forces must equal the vehicle mass times its acceleration in both the vehicle directions, and the moment of those forces about the vehicle mass center must be equal to the product of the yaw acceleration and the vehicle yaw moment of inertia. Thus, the main objective of the vehicle model is to accurately predict tire shear forces.

Primary factors influencing vehicle system motions. A vehicle is also subject to aligning moments in the tire contact patches. The aligning moment has a negligible direct effect on the vehicle yaw, but, due to steering compliance, it can be a significant factor in determining the all-important shear forces. Another behavior that influences the vehicle response involves the rotary motion of the car body in roll and pitch. Mechanical energy is transferred as the vehicle pitches and rolls, and these motions contribute to the vehicle transient response. Besides the tire/road interactions, the only forces and moments acting on the vehicle are due to aerodynamic effects. They have a secondary influence, but are relatively easy to add to multibody models.

The model is based on a rigid body that represents the main body of the vehicle and has six DOF. An additional four bodies are added, each with a single translational DOF, to account for the vertical movements allowed by the suspensions. The wheel bodies are positioned such that the origins of their local coordinate systems are nominally at the locations of the centers of tire. Longitudinally, the origins of the front and rear wheels are separated by the vehicle wheelbase. Laterally, they are separated by the vehicle front and rear track widths.

The user of a vehicle model must provide mass and inertia parameters for the bodies in the model. For the wheel bodies, one may set the moments of inertia to zero, and locate the mass centers at the wheel centers, nominally a height above the ground. The mass of each wheel body should be set to that portion of vehicle mass supported by the tire that is considered to move with the wheel. This value is commonly called the unsprung mass, and usually includes some of the mass of the suspension elements. The mass of the main body, called the sprung mass, must be set to the mass of the entire vehicle minus the unsprung masses.

Suspension springs, dampers, bump stops, and anti-sway bars affect movement of a wheel along the line of motion allowed by the suspension kinematics. In each case, some of the force generated by a component (e.g., a spring) acts to move the wheel, affecting the transfer of mechanical energy to and from the sprung mass. In addition, some of the force is reacted at other points or in other directions that do not move and therefore cannot affect the transfer of mechanical energy.

The effect of a suspension component at the wheel is calculated in three steps: multiply the suspension compression (measured at the wheel) by the kinematics ratio to determine the compression at the component, apply a known functional relationship (e.g., spring force vs. compression) to determine the force generated by the component, and multiply the component force by the kinematics ratio to obtain an effective vertical force at the wheel.

In general, different geometric ratios are needed for the suspension spring, the damper, and the bump stop. Different ratios are also used for front and rear, but the same ratios are used between left and right wheel on the same axle. The effect of the anti-sway bar is modeled with a linear spring between the two wheels linked by the bar. The two points in the add-line-force command are on the two wheels, the direction of the force is vertical force, and the magnitude is a spring rate multiplied by the vertical movement difference between the two points.

The primary challenge in developing a valid vehicle simulation model is to accurately predict the tire forces. Although the multibody program handles the kinematics and dynamics of the rigid bodies in the system, it is usually necessary to use an external (user defined) routine to compute tires forces and moments based on kinematics inputs. Many tire models (algorithms) exist for calculating tire forces [10], and most require the same inputs: vertical load, longitudinal slip, lateral slip, and inclination angle. As outputs, they calculate longitudinal force, lateral force, and aligning moment. To properly determine the tire forces and factor them into the vehicle model, it is necessary to define a point where the tire forces act on the multibody model; determine an expression for the vertical tire force that is required as an input for the tire model; establish the vector directions for the X and Y components of the tire shear force relative to the vehicle model; determine expressions for the kinematical inputs required by most tire models (longitudinal slip, lateral slip, and inclination angle); and use a tire model to determine the magnitudes.

Tires develop shear forces in response to deformation of the tire structure. The forces do not develop instantly, but build as the tire rolls. Researchers have found that the dynamic delay of the forces is primarily linked to the spatial distance covered by the tire [11]. The earliest approximation of this behavior was to treat the delay as a first-order lag. The characteristic parameter is called relaxation length, and is similar to a time constant, except that it has units of length rather than time. The response to steering is delayed sufficiently that the lag interacts with the dynamics of the vehicle system at low speed [12]. The lag for longitudinal slip is usually neglected.

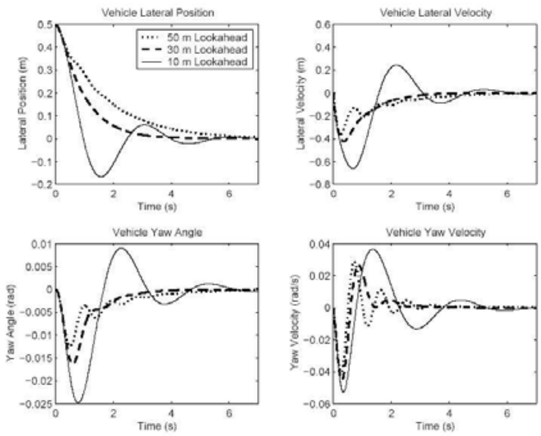

Figure 6- Yaw behavior.

Figure 7 and Figure 8 show how the eigenvalues shift for an oversteering and understeering car as the virtual force is shifted from 0:5m behind the neutral steer point to 0:5m in front of the neutral steer point. The square denotes the initial position behind the neutral steer point. The eigenvalues verify that the system is unstable when the application point is behind the neutral steer point. As the virtual force is shifted forward, the system becomes stable, but as the force is moved further forward the system becomes oscillatory and eventually unstable.

This instability is due to the lack of lookahead in the system. Interestingly, in the case discussed in this section, the force is shifted to the neutral steer point but the sensing location remains at the CG Therefore, in the oversteering case the sensing location is actually behind the control force while in the understeering vehicle it is slightly in front of this control force. In either case, with hardly any lookahead the system response is excellent. The limitation is that the neutral steer point is only marginally stable, yielding an eigenvalue at zero. If the control force is moved slightly rearward, the system will become unstable. It is unlikely that the vehicle parameters can be known with the accuracy necessary to pin point this location.

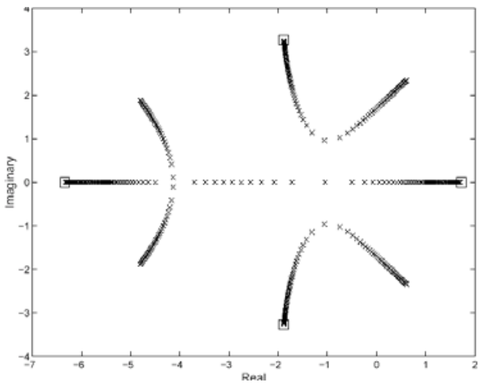

Figure 7- Eigenvalues for oversteering vehicle.

Even variations in vehicle loading or tire pressure can shift this point and create instability. To be robust to parameter uncertainties the control force should be shifted in front of the neutral steer point. As this force is moved towards the front axle of the vehicle, the damping lowers and a critical speed exist. The system response can be improved by incorporating an appropriate amount of lookahead. The role of lookahead in vehicles with the control force at the front axle (corresponding to having only steering) is well known but with the ability to shift the force there are two different variables that influence stability and system behavior.

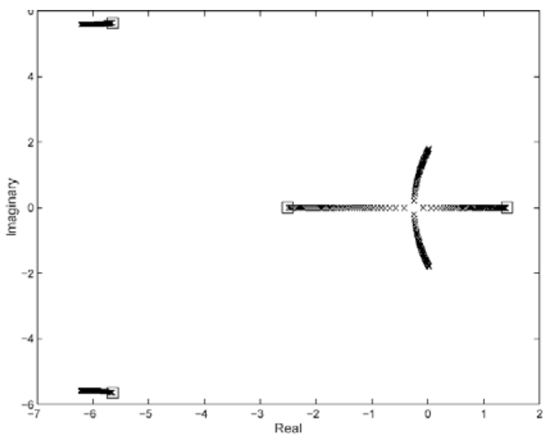

Figure 8- Eigenvalues for understeering vehicle.

CONCLUSIONS

Considering the development methodology, the prototype project achieves competition proposals. The benchmark was essential to keep the design near the competitive vehicles. The vehicle concept chouse phase had raised importance in the prototype developed since innovate sufficiently to had been used with benefits of total performance. The used references and computational tools had supplied the necessary support so that the sizing and design details could occur in a coherent form with engineering principles. The test evaluation was agreed perfect with the simulation models. A well-engineered suspension system does not automatically make a fast race car. Although this paper has concentrated on the design aspect, development is just as important to the success of the package. Because the design process must take place within a given time constraint, the first suspension design might not provide the best handling. It is not uncommon to make design changes after the car is completed. It is more important for FSAE teams to compromise on the overall design so that the car can be completed and tested prior to competition.

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