
Numerical Design of Racecar Suspension Parameters

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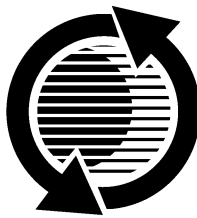
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ABSTRACT

Even with the rapidly evolving computational tools available today, suspension design remains very much a black art. This is especially true with respect to road cars because there are so many competing design objectives. In a racecar some of these objectives may be neglected. Even still, just concentrating on maximizing road-holding capability remains a formidable task. This paper outlines a procedure for establishing suspension parameters, and includes a computational example that entails spring, damper, and anti-roll bar specification. The procedure is unique in that it not only covers the prerequisite vehicle dynamic equations, but also outlines the process that sequences the design evolution. The racecar design covered in the example is typical of a growing number of small open wheel formula racecars, built specifically for American autocrossing and British hillclimbs. These light-weight racecars, 250-300 kilograms, often employ motorcycle engines producing in excess of 75 kilowatts. The power to weight ratio rivals that of many high level formula racecars. Due to the nature of the application, braking and cornering performance is equally impressive. The model presented embraces the latest trends with respect to racecar vehicle dynamics. Special emphasis, including discussion of theory and analysis, is placed on damper specification.

OVERVIEW

There are four vehicle dynamic modal characteristics that need to be considered when designing a road car or a racecar [1]:

1. Pitch: The vehicle rotation about the y-axis
2. Roll: The vehicle rotation about the x-axis
3. Heave: Uniform rectilinear motion along the z-axis of each tire contact patch
4. Warp: Non-uniform rectilinear motion along the z-axis of each tire contact patch

These vehicle dynamic characteristics are largely defined by the ride rates and roll rates of each axle. Key design parameters are appropriately set forth. Ideal road-holding is achieved when the unsprung mass ride motion relative to the road surface is zero. This occurs when the unsprung mass follows the contour of the road exactly [2]. While this is not achievable, minimizing this relative motion is very desirable. To accomplish this objective, the suspension engineer would want to select the lowest possible spring rates. However, there is a conflicting simultaneous objective of controlling the ride motion of the sprung mass relative to the road. To control the sprung mass motion it is tempting to specify very high spring and damper rates. Optimization of these two conflicting objectives is resolved empirically. The body bounce frequency is a parameter, consistent for varying masses, that characterizes this compromise. Road cars, including family sedans and sports cars, have body bounce frequencies ranging from 0.8 to 1.5 Hertz. For non-aero formula cars the optimal body bounce frequency is typically found to be around 2 Hertz, whereas aero cars can be as high as 5-7 Hertz. The rear axle frequency is generally slightly higher to reduce the vehicle's pitch tendency that occurs when the vehicle encounters a road surface discontinuity. There is a logarithmic relationship between ride frequency and static wheel deflection ($f_n = 5/x^{0.5}$, where deflection is in cm). For a natural frequency of 2 Hertz, static deflection is approximately 6 centimeters. Given the ride frequency parameter, ride rates are easily derived once sprung corner weights (or masses) are determined. The equation relating body bounce frequency, ride rates, and sprung mass, is very simple yet often overlooked in lower formula level auto racing competition. The ride rate is the effective rate of suspension and tire springs in series, or the body rate with respect to ground. The implications of the ride rates derived from the equation affect pitch, roll, heave, and warp. The suggested rates provide a good starting point to achieve tolerable pitch, roll and heave, and sufficient warp. It is also significant to note that the vehicle's road-holding ability is maximized when the wheel loads are maintained constant and proportional to the tire's adhesion capacity. While this observation suggests another

idealized impossibility (because weight transfer is a consequence of vehicle operation), implications do exist for the vehicle control strategy devised by the suspension engineer, and for the driver of the vehicle.

Since weight transfer is a function of vehicle mass, acceleration, CG height, and track width or wheel base length, the suspension designer must manage the effects of it as best possible. While designing the roll axis to pass through the CG would virtually eliminate body roll, the result would be instantaneous weight transfer. It is far better to make the roll moment as large as it can be effectively managed. In doing so, the effective rate of weight transfer can be reduced. Body roll is only recently being embraced as an essential element in creating better handling vehicles. Ford's SVT Mustang is a notable example. Previously, high performance road cars and racecars have typically been setup to minimize body roll. This is largely believed to be the result of attempts at masking poor suspension kinematics. If suspension kinematics do not afford adequate camber control throughout suspension travel, then softer roll rates will degrade the road-holding capability of the vehicle. This situation has been historically so prevalent that roll control, or more specifically the minimization of roll, is directly associated with good handling by the majority of automotive enthusiasts, racers and journalists.

Warp is the suspension's ability to comply with road surface aberrations. If this compliance were unnecessary, cars would be manufactured without suspensions. It has been common for racers to effectively eliminate their suspensions by stiffening them excessively in efforts to reduce pitch, roll, heave, as well as the aforementioned unwanted camber gain or loss. This naturally is done so at the expense of warp. By not providing adequate warp, the dynamic load variations increase, and cornering performance degrades.

Springs are employed to absorb shock that would otherwise be fully and immediately transferred to the chassis when the vehicle encounters a road surface discontinuity. The reason springs are used instead of just having a flexible chassis facilitate warp, is because the flexible chassis is essentially an undamped spring. The shock absorbers, more appropriately referred to as dampers, are there primarily to control the release of the energy stored in the springs (they also control bump energy and provide roll damping). If spring energy were not managed through the use of dampers, the energy release would be uncontrolled. Oscillation of both the sprung mass and unsprung mass would result. The oscillation, or bouncing, of these masses, would create stability and vehicle control issues.

While suspension tuning can enhance the handling of any suspension type, four-wheel independent suspensions are preferred because they inherently possess fewer compromises with respect to vehicle dynamics. Similarly, double wishbones are preferred over struts for

superior wheel control. Double wishbones are selected for the open wheel racecar that is covered in the sample calculations. While suspension geometry is not specifically covered in this paper, the kinematic goals are to minimize the affects due to bump, roll, and steering throughout the entire suspension travel with respect to the ideal wheel position of being perfectly upright and in continuous contact with the road surface. The double wishbone suspension, especially with inboard spring/dampers, represents an elegant solution for wheel location, in that virtually all the members can be placed in perfect tension or compression. Consequently, the design is very weight efficient, and thus popular in racing applications.

In this paper's design example, the spring rates suggested by the body bounce frequency equation were deemed to be marginally low in accommodating the anticipated road loads. Consequently, an inboard coil over damper suspension design was specified to accommodate the kinematics for progressive wheel rates. With the design, wheel rates at trim height can be made more appropriate without designing in excessive wheel travel that would compromise the suspension geometry. The suspension design incorporates a bell-crank actuated inboard coil over damper assembly. For analysis, the progressive rates of the suspension are neglected. The rates at trim height represent a worse case analysis. As a side note regarding progressive rate suspensions, there are downsides to such a design. Most notably, rates of the front and rear changing in opposing directions as the vehicle pitches. Depending on the magnitude, this can potentially upset the balance of the vehicle. If progressive rates are used, it is advisable to make the rates of progression symmetrical, front to rear. This will help diminish the likelihood of adversely affecting the vehicle's balance during roll. Also, the rates of progression should not be overly aggressive. For example, motion ratios of 0.9:1 at full droop to 1.4:1 at full jounce, may represent a reasonable rate of progression.

The Installation Ratio (IR), also referred to as the Motion Ratio (MR), is the change in spring length with respect to vertical wheel movement, or spring displacement over wheel displacement [3]. While the MR is typically reverse engineered based on the spring/damper's package size and rates available, it is reasonable to start with a MR of 1:1 for the type of racecar discussed here. With this ratio, the damper stroke will be equal to the vertical wheel displacement (suspension travel). However, as suggested, the ratio is usually manipulated to accommodate the spring/damper package. Clearly, the ratio for a given spring/damper assembly is optimized when the maximum wheel travel utilizes the entire spring/damper travel available. When using undersized spring/dampers, low speed damper resolution may become a concern. For the design exercise that follows, the installation ratio at trim height is 1:1. So at trim, the spring rates and wheel rates are identical.

The final suspension parameters to be specified are the damper rates. Dampers are the least well understood suspension components. There does not exist any text book formulas to produce damper specifications which will ensure appropriate damping characteristics. Consequently, damper specification remains an esoteric area of vehicle dynamics. It is only recently that racers are realizing how significant damping characteristics can be. During the early 90's significant insight to damper possibilities were uncovered in F1 through the use of active suspensions. With active suspensions, decoupling of the modal characteristics became possible. Designers naturally were unwilling to give up the performance gains when active suspensions were banned after the 1993 season. Developments in damper specification are occurring rapidly in all levels of motorsport. While there is increased emphasis on damping strategy, information regarding recent innovations is not readily available due to the intensely competitive nature of the development source. What appears in the calculations section of this paper, is a strategy where wheel motion is to be controlled through bump, or compression, damping, and body motion is controlled through rebound damping. Rebound/compression ratios for production street cars and racecars typically vary from 1.5:1 to 4:1. These ratios are also presented in the form of 60/40 to 80/20, respectively. A 3:1 ratio has historically been considered optimal, and remains a very prevalent ratio. Analytically, damping rates are often specified as a percent of critical damping. Unfortunately, the ideal damping rates for a vehicle are specific to the operating environment. Since it is extremely difficult to accurately anticipate the range of operating conditions, the task of specifying the optimal compromise is just as daunting. These rates continue to be empirically derived. Racers develop data bases regarding track conditions and try to continually improve upon their best prior effort. For the sample calculations, rates are specified for operating conditions typical of an American autocross, British hillclimb, or club level road course.

DESIGN PROCEDURE

1. Establish Vehicle Parameters (size, weight & power)
2. Specify Suspension Package
3. Specify Ride Frequencies & Ride Frequency Ratio
4. Estimate Sprung & Unsprung Corner Weights
5. Derive Ride, Suspension, & Spring Rates
6. Derive Initial Roll Rates without Anti-roll bars & Compute Wheel Displacement at Maximum Cornering Loads
7. Calculate Lateral Load Transfer Distribution (LLTD) without Anti-Roll Bars
8. Specify Anti-roll Bars to Produce Desired Roll Rates & LLTD
9. Specify Damper Rates

Racecars are typically designed from the outside-inward. That is to say, given the basic layout – size, weight & powertrain, tires/wheels are specified first, followed by uprights and control arms with attachment points. Once the tire/wheel package and suspension type (i.e. coil-over, strut, inboard/outboard) are specified, reasonable estimates of sprung and unsprung weights are used to determine ride rates. Initial ride rates, rate of chassis with respect to ground, should be derived from ride or body bounce frequency equations. Ride frequencies are influenced by the spring rates of both the suspension and the tires. Obtain tire data available through the manufacturer. This data is critical to the design, so use the best available information for the most likely operating condition (rim width, tire pressure, camber angle, load and velocity). With ride rates and tire rates determined, suspension rates (chassis with respect to the wheel) can now be calculated. The kinematics of the suspension must be specified in order to determine the actual spring rates. Once relative motion ratios of the spring vs. the wheel are established, the desired spring rates can be determined. Since springs are typically made available with specified rates, a spring close to the calculated value will be selected. Suspension rates, ride rates and frequencies are consequently affected. So recalculate those values and determine the sensitivity of the system based on the available spring rates.

The next step involves the determination of the roll rates for each axle without anti-roll bars (ARB). Roll stiffness is calculated to determine the roll rate or roll flexibility. Roll flexibility, specified in degrees/g, is used to determine the maximum wheel displacement due to chassis roll with respect to the wheels. Wheel displacement at the maximum anticipated cornering load is determined. This displacement is compared to available suspension travel. If sufficient suspension travel exists with these initial roll rates, a single ARB will be added to either the front or the rear suspension to balance the car. In the event the car is balanced without an ARB, it is recommended that the smallest (lightest) possible ARB be incorporated in the suspension to provide a convenient means of track-side chassis tuning.

Calculation of the lateral load transfer distribution (LLTD) requires values of roll stiffness and roll flexibility, as well as front and rear roll center heights, weight distribution and an estimate of total vehicle weight with driver. Roll center heights are dependent on the suspension geometry. They represent an idealized concept where the sprung mass rotation is about the roll axis. The roll axis is defined as the line that passes through the front and rear roll centers. With four-wheel independent suspensions, the roll center heights can be readily manipulated to produce the desired front and rear roll moments. Typically on an open wheel formula car, the front roll center is very near the ground, where the rear roll center is slightly higher, both with respect to ground. So the front roll center will be near zero, positive or negative, and the rear roll center height will be slightly more positive. The resulting roll moments have been found to produce a reasonable

compromise with respect to roll moments of manageable magnitude. The LLTD calculated without anti-roll bars is compared to the desired LLTD. For a racecar the desired LLTD will need to be less than one to achieve neutral handling. This is because there are several factors producing understeer that are not taken into account by the LLTD equation. Examples being, tire cornering stiffness, camber thrust, roll steer, lateral force compliance steer, aligning torque, lateral load transfer, and steering system compliance [2]. These considerations are beyond the scope of this paper, but are mentioned for insight into the specification of a LLTD less than one. Specify a front or rear anti-roll bar to produce the desired LLTD. Recalculate the roll rate and determine the vertical wheel displacement. If the resulting roll rate is acceptable, then a second bar is not necessary. If adding a second bar, increase the first bar proportionally to that of the second bar to maintain the desired LLTD.

The damper specification begins with accurate estimates of the sprung and unsprung masses for front and rear corners of the vehicle. Quarter car models are used in this analysis. Since tire and suspension rates have already been established, critical wheel and critical body damping values can now be calculated. These values are with respect to vertical wheel displacement. So the installation ratio will need to be taken into account when establishing the damper specification. Even at the highest level, damping rates for a given application are still optimized using an iterative process. This is most evident by the number of springs and dampers high level racing teams bring to practice test sessions. What is suggested here will get you close, but ultimately operating conditions and driver preferences will determine what will provide the optimal compromise as measured by a stopwatch. Specify compression damping as a percent of critical wheel damping, say 20-50%, and use rebound damping to control body motion. For the most part, excessive rebound damping, provided it is balanced front to rear, is not likely to be a problem in a racecar application. In addition to the general stabilizing effect on body motion, high rebound damping rates help keep the vehicle CG low during corner entry. This is especially beneficial to vehicles with marginal camber control. Try to be close to the 3:1 rebound/compression ratio. Accept values of critical body damping in the range of 70-140%. Note that the rebound/compression ratio is an absolute value, where the percent of critical damping is with respect to two different values. Ideally, two-way adjustable shocks would be specified to cover both respective ranges; however, not all racing budgets can accommodate such expenditures.

CALCULATIONS

1. SPECIFY TIRES/WHEELS:

	<u>Front</u>	<u>Rear</u>
Tire Size	20.0x7.0-13 in	20.0x8.0-13 in
Tire Mass	3.99 Kg	4.08 Kg
Tire Camber	-3.0 Degrees	-2.0 Degrees
Tire Pressure	90 kPa	90 kPa
Tire Stiffness	100 N/mm	125 N/mm
Wheel Size	13x7.0 in	13x8.0 in
Wheel Mass	3.35 Kg	3.60 Kg

Also known at time of tire specification:

Wheel base	1.88 m
Track width front	1.27 m
Track width rear	1.24 m
Vehicle mass w/driver (est.)	300-320 Kg
Power (est.)	70-80 kW
Suspension travel front, jounce/droop	5 cm/5 cm
Suspension travel rear, jounce/droop	5 cm/5 cm

2. SPECIFY SUSPENSION PACKAGE:

Front:	Double wishbone, pushrod actuated inboard coil over damper, outboard disc brakes
Rear:	Double wishbone, pushrod actuated inboard coil over damper, inboard disc brakes

3. SPECIFY RIDE FREQUENCIES AND RIDE FREQUENCY RATIO

Front ride frequency, f_{nr} :	2.0 Hz
Rear ride frequency, f_{nr} :	2.2 Hz
Ride frequency ratio, f_{nr}/f_{nr} :	1.1

4. ESTIMATE SPRUNG AND UNSPRUNG CORNER WEIGHTS:

	<u>Front</u>	<u>Rear</u>
Unsprung mass:		
Tire	4.0 Kg	4.1 Kg
Wheel	3.4	3.6
Uprights	1.2	0.8
Hubs	1.2	0.9
Rotors	0.6	---
Calipers (w/linings)	1.0	---
Suspension Links	1.6	1.5
Drive Shaft	---	1.5
Unsprung Mass, m:	13.0 Kg	12.4 Kg
Sprung Mass w/driver, M:	60.0 Kg	70.0 Kg
Corner Mass, m + M:	73.0 Kg	82.4 Kg
Total Vehicle Weight	3049 N	

Given a 47/53 weight distribution,

$$W_f = 1433 \text{ N} \quad W_r = 1616 \text{ N}$$

5. DERIVE RIDE, SUSPENSION, AND SPRING RATES:

- Ride rate, rate of chassis with respect to ground, K_R (N/m)

$$f_{nf} = \frac{1}{2\pi} \sqrt{\frac{K_{Rf}}{M}} = 2.0\text{Hz} \quad f_{nr} = \frac{1}{2\pi} \sqrt{\frac{K_{Rr}}{M}} = 2.2\text{Hz}$$

Where: f_n = Body bounce frequency (cycles/sec)
 K_R = Ride rate (N/m)
 M = Sprung corner mass ($\text{Kg} = \text{N} \cdot \text{sec}^2/\text{m}$)

$$K_R = (2\pi \cdot f_n)^2 \cdot M$$

$$K_{Rf} = (2\pi \cdot 2.0 \text{ sec}^{-1})^2 \cdot 60 \text{ N} \cdot \text{sec}^2/\text{m} = 9475 \text{ N/m}$$

$$K_{Rr} = (2\pi \cdot 2.2 \text{ sec}^{-1})^2 \cdot 70 \text{ N} \cdot \text{sec}^2/\text{m} = 13375 \text{ N/m}$$

- Suspension rate or suspension stiffness, rate of body with respect to wheel, K_S (N/m)

$$K_R = K_S \cdot K_T / (K_S + K_T) \Rightarrow 1/K_R = 1/K_T + 1/K_S$$

$$1/K_S = 1/K_R - 1/K_T$$

$$1/K_{Sf} = 1/9475 \text{ N/m} - 1/100000 \text{ N/m} = 1/10467 \text{ N/m}$$

$$K_{Sf} = 10467 \text{ N/m}$$

$$1/K_{Sr} = 1/13375 \text{ N/m} - 1/125000 \text{ N/m} = 1/14978 \text{ N/m}$$

$$K_{Sr} = 14978 \text{ N/m}$$

- Spring rates, K_{SPR} (N/m)

For constant linkage ratio,

$$K_S = K_{SPR} (IR)^2$$

Where: IR = Installation Ratio, spring motion over wheel motion (unitless). Also referred to as Motion Ratio (MR).

If IR = 1, then $K_{SPR} = K_S$

6. DERIVE ROLL RATES:

- Roll stiffness of suspension, K_ϕ (Nm/rad):

$$K_\phi = 0.5 K_S t^2 \text{ Where: } t = \text{track width (m)}$$

$$K_{\phi f} = 0.5 \cdot 10467 \text{ N/m} \cdot 1.27^2 \text{ m}^2 = 8441 \text{ Nm/rad}$$

$$K_{\phi r} = 0.5 \cdot 14978 \text{ N/m} \cdot 1.24^2 \text{ m}^2 = 11515 \text{ Nm/rad}$$

- Roll rate, or roll flexibility, R_ϕ (deg/g):

$$R_\phi = (W/g) \cdot h_i / (K_f + K_r - Wh_i)$$

Where: W = Total vehicle weight with driver
 $h_i = h - h_{ra}$ = distance of roll axis from CG

Given: h = CG height = 300 mm
 h_f = front roll center height = 0 mm
 h_r = rear roll center height = 10 mm
 h_{ra} = roll axis height at CG = 5.3 mm

$$h_i = h - h_{ra} = 0.3 \text{ m} - 0.0053 \text{ m} = 0.295 \text{ m}$$

$$R_\phi = 3049 \text{ N/g} \cdot 0.295 \text{ m} / (8441 \text{ Nm/rad} + 11515 \text{ Nm/rad} - 3049 \text{ N} \cdot 0.295 \text{ m})$$

$$R_\phi = 0.0472 \text{ rad/g} = 2.70 \text{ deg/g}$$

Given a maximum anticipated lateral acceleration of 1.4g, wheel displacement (Δz_w) due to roll (ϕ) is:

$$\phi = 1.4g \cdot 2.70 \text{ deg/g} = 3.78 \text{ degrees}$$

$$\Delta z_w = 0.5 t_f \cdot \sin \phi = 0.5 \cdot 1.27 \text{ m} \cdot \sin 3.78^\circ = 0.0419 \text{ m}$$

Since suspension travel in bump is 5.0 cm, this is tolerable as is. Based on this calculation a single anti-roll bar will be sufficient.

7. CALCULATE LLTD WITHOUT ANTI-ROLL BARS:

$$\text{LLTD} = \Delta F_{zf} / \Delta F_{zr} = [K_{\phi f} R_\phi + h_f \cdot (W_f/g)] / [K_{\phi r} R_\phi + h_r \cdot (W_r/g)]$$

Given: $K_{\phi f} = 8441 \text{ Nm/rad}$ $K_{\phi r} = 11515 \text{ Nm/rad}$
 $R_\phi = 0.0472 \text{ rad/g}$ $R_\phi = 0.0472 \text{ rad/g}$
 $h_f = 0.00 \text{ m}$ $h_r = 0.01 \text{ m}$
 $W_f = 1433 \text{ N}$ $W_r = 1616 \text{ N}$

LLTD without anti-roll bars,

$$\text{LLTD} = \frac{8441 \text{ Nm/rad} \cdot 0.0472 \text{ rad/g} + 0.00 \text{ m} \cdot 1433 \text{ N/g}}{11515 \text{ Nm/rad} \cdot 0.0472 \text{ rad/g} + 0.01 \text{ m} \cdot 1616 \text{ N/g}}$$

$$\text{LLTD}_{w/o \text{ ARB}} = \frac{398.4 \text{ Nm/g} + 0 \text{ Nm/g} = 0.721 \text{ (unitless)}}{543.5 \text{ Nm/g} + 16.16 \text{ Nm/g}}$$

8. SPECIFY ANTI-ROLL BARS TO PRODUCE DESIRED ROLL RATES AND LLTD:

- Adding front ARB will reduce R_ϕ and increase $K_{\phi f}$. Assume R_ϕ does not change and solve for desired LLTD:

$$\text{Given LLTD}_{\text{desired}} = 0.85$$

$$K_{\phi f w/ARB} \cdot R_\phi = 0.85 \cdot (543.5 \text{ Nm/rad} + 16.16 \text{ Nm/rad})$$

$$K_{\phi f w/ARB} = 475.7 \text{ Nm/g} / 0.0472 \text{ rad/g} = 10079 \text{ Nm/rad}$$

$$K_{\phi f \text{ ARB}} = K_{\phi f w/ARB} - K_{\phi f w/o \text{ ARB}}$$

$$K_{\phi f \text{ ARB}} = 10079 \text{ Nm/rad} - 8441 \text{ Nm/rad} = 1638 \text{ Nm/rad}$$

- Recalculate roll rates with Anti-roll bar:

$$R_\phi = (W/g) \cdot h_i / (K_f \phi + K_r - Wh_i)$$

$$\text{Where: } K_{\phi f \phi} = K_{\phi f w/ARB} = 10079 \text{ Nm/rad}$$

$$K_{\phi r} = 11515 \text{ Nm/rad}$$

$$R_\phi = 3049 \text{ N/g} \cdot 0.295 \text{ m} / (10079 \text{ Nm/rad} + 11515 \text{ Nm/rad} - 3049 \text{ N} \cdot 0.295 \text{ m})$$

$$R_\phi = 0.0434 \text{ rad/g} = 2.49 \text{ deg/g}$$

Wheel displacement (Δz_w) due to roll (ϕ) is:

$$\phi = 1.4g \cdot 2.49 \text{ deg/g} = 3.48 \text{ degrees}$$

$$\Delta z_w = 0.5 t_f \cdot \sin \phi = 0.5 \cdot 1.27 \text{ m} \cdot \sin 3.48^\circ = 0.0385 \text{ m}$$

9. DAMPER SPECIFICATION

Critical body damping values:

$$C_{cr_b} = \sqrt{4K_S M}$$

$$C_{cr_{bf}} = \sqrt{4(10467 \text{ N/m})(60 \text{ N s}^2/\text{m})}$$

$$C_{cr_{bf}} = 1585 \text{ N s/m}$$

$$C_{cr_{br}} = \sqrt{4(14978 \text{ N/m})(70 \text{ N s}^2/\text{m})}$$

$$C_{cr_{br}} = 2048 \text{ N s/m}$$

Critical wheel damping values:

$$C_{cr_w} = \sqrt{4(K_S + K_T)m}$$

$$C_{cr_{wf}} = \sqrt{4(10467 \text{ N/m} + 100000 \text{ N/m})(13.0 \text{ N s}^2/\text{m})}$$

$$C_{cr_{wf}} = 2397 \text{ N s/m}$$

$$C_{cr_{wr}} = \sqrt{4(14978 \text{ N/m} + 125000 \text{ N/m})(12.4 \text{ N s}^2/\text{m})}$$

$$C_{cr_{wr}} = 2635 \text{ N s/m}$$

Damper specification:	Front	Rear
Rebound/compression ratio	3:1	3:1
Specified compression damping	719 Ns/m	790 Ns/m
Percent of critical wheel	30%	30%
Percent of critical body	45%	39%
Specified rebound damping	2157 Ns/m	2370 Ns/m
Percent of critical body	136%	116%

CONCLUSION

The procedure outlined is intended to produce a reasonable baseline. Testing the package will always produce the best results. With an inboard suspension, adjustable links provide an effective means to try different suspension rates. If dampers are adjustable in compression or rebound, manufacturers typically have instructions for adjusting or tuning their product. Following the procedures outlined will help avoid the most common causes of traction loss, which are overly stiff springs and overly damped suspensions. The later is much less obvious and consequently much more difficult to determine.

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