7 DOF Vehicle Model for Understanding Vehicle Fluctuation During Straight Running

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ABSTRACT

This paper concerns an introduction on how to lead the 7 degrees of freedom (7DOF) model for understanding the vehicle-fluctuation during straight running. Despite the steering wheel being fixed at on-center by anchor rod connected to the floor, vehicle run with a very slow lateral fluctuation, of which amplitude is small but becomes maximum at a certain speed zone, was observed by a vehicle test on a flat road of the proving ground. For understanding this phenomenon, I provided the motion equation of the 7DOF model. By analysis using the motion equation, the existence of a speed zone where the amplitude becomes maximum is confirmed. This appears when the rear compliance steer is the proverse compliance steer and disappears by changing it to the adverse compliance steer. Where, the adverse/proverse compliance steer mean the characteristics of compliance steer that increase/decrease the side-slip angle versus the vector direction of the vehicle (1).

INTRODUCTION

I have explained the existence of fluctuation during straight running caused by the compliance of steering/suspension system in SAE 2001-01-3433 (1) and confirmed the existence of a very slow yaw-peak of which frequency changes with vehicle speed by a vehicle test conforming to ISO/TR8726 in IMECE2001/DE-23259 (2). The slow yawing coming from the angular velocity change of steering and front suspension was found by an eigen vector analysis using the 7DOF model analysis. This paper describes the leading process of the 7DOF model equation whose description had been abridged because of the paper space limitation, and the existence of the speed zone where amplitude becomes maximum that appears when the rear compliance steer is the proverse compliance steer and disappears by changing it to the adverse compliance steer.

REVIEW OF SAE 2000-01-3433 & IMECE2001/DE-23259

A very low frequency vehicle fluctuation of which amplitude becomes maximum at 60 or 80 km/h is confirmed as shown in Fig.1 (1). The test was done by a large sized bus fixing the steering wheel on center with anchor rod for avoiding the influence of steering wheel input. Fig.2 (2) shows the yaw-gain tested in conformance with ISO/TR8726. The heave (low peak) is observed at the frequency zone from 0.1 to 0.3 Hz. This low frequency corresponds to that of Fig.1. In order to understand this low frequency and its speed-dependence characteristics, I provided the 7DOF vehicle model in considering the lateral and yawing motions of the sprung mass, front un-sprung mass, rear un-sprung mass and compliance at steering link, front suspension links and rear suspension links as shown in Fig.3 (2).

Fig.1 Vehicle fluctuation during straight running
MOTION EQUATION OF 7DOF MODEL

In Fig.3, the x-y is the coordinate of the sprung mass, the mass of which is \( M \) and the inertia is \( I \), moving at the speed \( V \) with the side slip angle \( \beta \), and yawing velocity \( \phi \).

The \( x_1-y_1 \) is the coordinate of the front un-sprung mass, the mass of which is \( M_F \) and the inertia is \( I_F \), deviating \( y_F \) and twisting \( \delta \) to \( x \)-axis caused by compliance spring/damping elements, \( K_{F1}/D_{F1} \) and \( K_{F2}/D_{F2} \) which exist in the suspension linkage connecting to the sprung mass, \( M \).

The \( x_3-y_3 \) is the coordinate of the steering system mass, the mass of which is \( M_s \) and the inertia is \( I_s \), moving at the speed \( V_s \) with the side slip angle, \( \beta_s \).

The lateral and rotational motion equations of sprung-mass are written as follows;

\[
\begin{align*}
M(\ddot{y}_y + v_y \phi) &= -F_{yF1} - F_{yF2} - F_{yR1} - F_{yR2} + F_{yST} \\
M(\ddot{y}_y + v_y \phi) &= +F_{yF1} \cos \delta_{CF} + F_{yF2} \cos \delta_{CF} - F_{yS} \\
I \ddot{\phi} &= -x_F F_{yF1} - x_F F_{yF2} - x_S F_{yST} - x_R F_{yR1} - x_R F_{yR2} \\
I \ddot{\phi} &= +x_F F_{yF1} + x_F F_{yF2} + x_S F_{yST} + x_R F_{yR1} + x_R F_{yR2} 
\end{align*}
\]

\((7-1)\)

The lateral and rotational motion equations of front sprung-mass are written as follows;

\[
\begin{align*}
M_F(\ddot{y}_1 + v_1 \phi_1) &= +F_{yF1} \cos \delta_{CF} + F_{yF2} \cos \delta_{CF} - F_{yS} \\
M_F(\ddot{y}_1 + v_1 \phi_1) &= -F_{yF1} \cos \delta_{CF} - F_{yF2} \cos \delta_{CF} + F_{yS} \\
I_F \ddot{\phi}_1 &= l_{FL} F_{yF1} \cos \delta_{CF} + l_{FL} F_{yF2} \cos \delta_{CF} - l_{FS} F_{yS} + T_{FS} \\
I_F \ddot{\phi}_1 &= -l_{FL} F_{yF1} F_{yF2} \cos \delta_{CF} + l_{FS} F_{yS} - T_{FS} \tag{7-2}
\end{align*}
\]

The lateral and rotational motion equations of rear sprung-mass are written as follows;

\[
\begin{align*}
M_R(\ddot{y}_2 + v_2 \phi_2) &= +F_{yR1} \cos \delta_{CR} + F_{yR2} \cos \delta_{CR} - S_{FR} \\
M_R(\ddot{y}_2 + v_2 \phi_2) &= -F_{yR1} \cos \delta_{CR} - F_{yR2} \cos \delta_{CR} + S_{FR} \\
I_R \ddot{\phi}_2 &= l_{RL} \cdot F_{yR1} + l_{RL} \cdot F_{yR2} - l_{RT} S_{FR} \\
I_R \ddot{\phi}_2 &= -l_{RL} \cdot F_{yR1} - l_{RL} \cdot F_{yR2} + l_{RT} S_{FR} = 0 \tag{7-6}
\end{align*}
\]

The lateral and rotational motion equations of steering system mass are written as follows;

\[
\begin{align*}
M_S(\ddot{y}_3 + v_3 \phi_3) &= +F_{yS} \cos \delta_{CS} + F_{yST} \cos(\delta_s + \delta_{CF}) - S_{FS} \\
M_S(\ddot{y}_3 + v_3 \phi_3) &= -F_{yS} \cos \delta_{CS} - F_{yST} \cos(\delta_s + \delta_{CF}) + S_{FS} = 0 \\
I_S \ddot{\phi}_3 &= l_{SL} F_{yS} \cos \delta_s + l_{SL} F_{yST} \cos(\delta_s + \delta_{CF}) - I_{FS} S_{FS} - T_{FS} \\
I_S \ddot{\phi}_3 &= -l_{SL} F_{yS} \cos \delta_s - l_{SL} F_{yST} \cos(\delta_s + \delta_{CF}) + I_{FS} S_{FS} + T_{FS} = 0 \tag{7-8}
\end{align*}
\]

Deploying the above equations, \((7-1),(7-2),\ldots,(7-7),(7-8)\), the matrix, \((7-55)\), is given as shown in the Appendix.
YAW-GAIN PEAK CONTROL STUDY BY 7DOF MODEL

Using the 7DOF model, parametric studies have been carried out. Fig.4 shows a calculated result in the case of the proverse compliance steer for the front and the proverse compliance steer for the rear. The specification is shown in Table 1 of the Appendix. There are two peaks. One is at 0.3 or 0.4 Hz and the other is at 2 or 3 Hz. It is considered that one comes from the natural frequency of sprung mass and the other comes from that of un-sprung mass. The numbers, 40, 60, 80, 100 show the vehicle speed in km/h. It is observed that the yaw-gain becomes maximum at 80 km/h in this case. Fig.5 shows a calculated result in the case of the proverse compliance steer for the front and the “adverse compliance steer for the rear”. Also, the specification is shown in Table 1 of the Appendix. The yaw-gain peak at 80 km/h has become lower and the peak height order has changed from 80, 100, 60, 40 km/h to 40, 60, 80, 100 km/h. Fig.6 shows, as a reference, a case where damping factors are added to the steering system and front suspension system of the case of Fig.4. In this case, the yaw gain peak order becomes 100, 80, 60, 40 km/h.

DISCUSSION

As the 7DOF model has been subjected to linearization boldly for finding the principle of fluctuation during straight running, we should understand that only the tendency and not the absolute value should be discussed when we use this 7DOF model. Based upon this understanding, the following discussion can be made. The existence of the speed zone where the amplitude becomes maximum is also confirmed by calculation as in Fig.4. This is back-up data to the experiment result shown in Fig.1. By changing the rear compliance steer from proverse to adverse, the tendency has changed as shown in Fig.5. This tendency appears more stable at higher speed zone than lower speed zone. The peak at 3 or 4 Hz in Fig.4 disappeared and then a cave appeared at 1 or 2 Hz zone. Judging from the eigen value analysis (Fig.7) and the eigen vector analysis (Fig.8) done in the previous paper\textsuperscript{2}, the 1 or 2 Hz corresponds to the root C that comes from the rear compliance and the 3 or 4 Hz corresponds to the root D that comes from the front compliance. Therefore the cave is considered to have come from the adverse compliance steer of rear suspension. By adding damping to the front suspension, the peak at 80 km/h in Fig.4 has been lowered as shown in Fig.6. In this case, the tendency shows that stability reduces more at higher speed zone than lower speed zone. This means that Fig.5 and Fig.6 have the opposite tendency as explained in SAE 2001-01-3433\textsuperscript{1}.
CONCLUSION

The leading process of 7DOF model is explained and examples of straight running analysis are shown. By constructing such a linear motion equation, it is possible to describe the bode diagram, to do eigen value analysis and eigen vector analysis. This is very useful to understanding the principle. As the existence of the characteristic roots are previously known by eigen value analysis, vehicle dynamics design focusing on the influence of the root becomes possible. By the eigen vector analysis, we can understand the constituents of the phenomenon. Vehicle dynamics design based on the principle is thus possible. The following have become known:

1. It is said generally that the vehicle stability will decrease with an increase of vehicle speed. However, strictly speaking, there are two other cases where decrease at the speed zone and increase occur simply with vehicle speed.

2. The cases due to the combination of front compliance and rear compliance. The case that decreases at a speed zone due to the larger front compliance under the combination of front proverse and rear proverse compliance steer. This disappears by changing the rear compliance from proverse to adverse, or increasing the damping of the steering and front suspension system.

ACKNOWLEDGMENTS

I developed this 7DOF model in 1997 as a part of my doctoral thesis, but did not include it in the final thesis because the vehicle model considering the unsprung mass effect did not meet with academic circles. I understood that further discussion would be necessary. I am thankful to SAE for giving me the opportunity to make this presentation, and also wish to thank the persons who show me their interest in my 7DOF model.

REFERENCES


APPENDIX

Here, I describe the continuous process referring to Fig.3, Fig.3-1 in Appendix, on how to deploy the fundamental equations led as (7-1),(7-2),(7-3),(7-4),(7-5),(7-6),(7-7) and (7-8) for getting the matrix, (7-55). Firstly the relation between the sprung-mass coordinate x-y, the un-sprung mass coordinates x1-y1 and x2-y2 is described, secondly the slip angle of tires is described, thirdly the side forces of tires, fourthly the forces applied between sprung mass and un-sprung mass, and then the process to reach the matrix (7-55) is described.

1-1. The relation between front un-spring mass and sprung mass

Fig.9 The relation between the coordinate x-y and the coordinates x',y'

General equation between the coordinate x-y and the coordinates x',y' is written as follows;

\[
\begin{align*}
    x &= x_1 \cos \delta_{CF} - y_1 \sin \delta_{CF} + x_F \\
    y &= y_1 \cos \delta_{CF} + x_1 \sin \delta_{CF} + y_F
\end{align*}
\]  
(7-9,10)

They coordinates of P_{PF1}, P_{PF2} and P_{PF3} are shown as follows;

\[
y_{PF1} = y_F + l_{F1} \sin \delta_{CF} \approx y_F + l_{F1}\delta_{CF} \\
y_{PF2} = y_F + l_{F2} \sin \delta_{CF} \approx y_F + l_{F2}\delta_{CF} \\
y_{PF3} = y_F + l_{F3} \sin \delta_{CF} + l_{S2} \sin \delta_{S} \approx y_F + l_{F3}\delta_{CF} + l_{S2}\delta_{S}
\]  
(7-11, 7-12, 7-13)

The relation between \(v_y\) and \(v_{y1}\) is shown as follows;

\[
\begin{align*}
    \dot{y}_F &= v_{y1} \cos \delta_{CF} - v_y - x_F \phi \\
    v_{y1} &= (\dot{y}_F + v_y + x_F \phi) / \cos \delta_{CF}
\end{align*}
\]  
(7-14-1, 7-14-2)

Here, we put \(\cos \delta_{CF} \approx 1\);

\[
y_{y1} = \dot{y}_F + v_y + x_F \phi
\]  
(7-15)

The relation between \(\phi, \phi_1, \delta_{CF}\) and \(\delta_{S}\) is shown as follows;

\[
\begin{align*}
    \dot{\phi}_1 &= \phi + \dot{\delta}_{CF} \\
    \dot{\phi}_3 &= \phi + \dot{\delta}_{CF} + \dot{\delta}_{S}
\end{align*}
\]  
(7-16, 7-18)

1-2. The relation between sprung mass and rear un-spring mass

Fig.10 The relation between the coordinate x-y and the coordinates x,y,

General equation between the coordinate x-y and the coordinates x',y' is written as follows;

\[
\begin{align*}
    x &= x_2 \cos \delta_{CR} - y_2 \sin \delta_{CR} + x_R \\
    y &= y_2 \cos \delta_{CR} + x_2 \sin \delta_{CR} + y_R
\end{align*}
\]  
(7-19,20)

They coordinates of P_{PF1} and P_{PF2} are shown as follows;

\[
y_{PF2} = y_R + l_{R1} \sin \delta_{CR} \approx y_R + l_{R1}\delta_{CR} \\
y_{PF3} = y_R + l_{R2} \sin \delta_{CR} \approx y_R + l_{R2}\delta_{CR}
\]  
(7-21, 7-22)

The relation between \(v_y\) and \(v_{y2}\) is shown as follows;

\[
\begin{align*}
    \dot{y}_R &= v_{y2} \cos \delta_{CR} - v_y - x_R \phi \\
    v_{y2} &= (\dot{y}_R + v_y + x_R \phi) / \cos \delta_{CR}
\end{align*}
\]  
(7-23-1, 7-23-2)

Here, we put \(\cos \delta_{CR} \approx 1\);

\[
v_{y2} = \dot{y}_R + v_y + x_R \phi
\]  
(7-24)

The relation between \(\phi, \phi_2, \delta_{CR}\) is shown as follows;

\[
\dot{\phi}_2 = \phi + \dot{\delta}_{CR}
\]  
(7-25)
2. THE SIDE SLIP ANGLE OF TIRES

2-1. The side slip angle of front tires

The side slip angle of front tire is described as follows;

\[ \beta_F = \tan^{-1}\left( \frac{v_{y3} + l_{FT} \dot{\phi}_F}{v_{x3}} \right) \]

\[ \approx \left( \frac{v_{y3} + l_{FT} \dot{\phi}_F}{v_{x3}} \right) \] (7-26)

Where, the \( v_{y3} \) is written as follows from the relation shown in Fig.9;

\[ v_{y3} = (\dot{y}_F + v_y + x_F \dot{\phi})\cos(\delta_{CF} + \delta_S) + l_{FS} \dot{\delta}_{CF} + l_{S2} \dot{\delta}_S \]

(7-27-1)

Here, we put \( \cos(\delta_{CF} + \delta_S) \approx 1 \);

\[ v_{y3} = \dot{y}_F + v_y + x_F \dot{\phi} + l_{FS} \dot{\delta}_{CF} + l_{S2} \dot{\delta}_S \]

(7-27-2)

Substituting Eq.(7-27-2) to \( v_{y3} \) of Eq.(7-26) and Eq.(7-18) to \( \dot{\phi}_F \) of Eq.(7-26) and putting \( v_{y3} \approx v_y \);

\[ \beta_F \approx \left[ \dot{y}_F + v_y + (x_F + l_{FT})\dot{\phi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S \right]/v_x \]

(7-26-1)

Rewriting the Eq.(2-26-1);

\[ \beta_F \approx \left[ \dot{y}_F + v_y + (x_F + l_{FT})\dot{\phi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S \right]/v_x \]

(7-26-2)

2-2. The side slip angle of rear tire

The side slip angle of rear tire is written as follows;

\[ \beta_R = \tan^{-1}\left( \frac{v_{y2} + l_{RT} \dot{\phi}_2}{v_{x2}} \right) \]

\[ \approx \left( \frac{v_{y2} + l_{RT} \dot{\phi}_2}{v_{x2}} \right) \] (7-28)

Substituting Eq.(7-24) to \( v_y \) of Eq.(7-28) and Eq.(7-25) to \( \dot{\phi}_2 \) of Eq.(7-28) and putting \( v_{y2} \approx v_y \);

\[ \beta_R = \left[ \dot{y}_R + v_y + x_R \dot{\phi} + l_{RT} \dot{\phi} + l_{RT} \dot{\delta}_{CR} \right]/v_x \]

\[ = \left[ \dot{y}_R + v_y + (x_R + l_{RT})\dot{\phi} + l_{RT} \dot{\delta}_{CR} \right]/v_x \] (7-29)

3. THE SIDE FORCES OF TIRES

The side force of front tire is described as follows;

\[ SF_F \approx FC_F \beta_F \]

\[ = FC_F \left( \dot{y}_F + v_y + (x_F + l_{FT})\dot{\phi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S \right)/v_x \]

(7-30)

The side force of rear tire is described as follows;

\[ SF_R \approx RC_F \beta_R \]

\[ = RC_F \left( \dot{y}_R + v_y + (x_R + l_{RT})\dot{\phi} + l_{RT} \dot{\delta}_{CR} \right)/v_x \]

(7-31)

4. RESTORING TORQUE, \( T_{FS} \) AT THE CONNECTING POINT, \( P_{FS} \)

Here, we put the torsional spring constant as \( K_{FS} \) and the damping coefficient as \( D_{FS} \), then \( T_{FS} \) is described as follows;

\[ T_{FS} = + K_{FS} \dot{\delta}_S + D_{FS} \cdot \dot{\delta}_S \]

(7-32)

5. THE FORCES APPLIED BETWEEN SPRUNG MASS AND UN-SPRUNG MASS

The front un-sprung mass is connected at \( P_{F1} \) and \( P_{F2} \) with \( x_{F1} \) and \( x_{F2} \) of sprung mass and also at \( P_{F1} \) with steering force input point \( x_s \). And, the rear un-sprung mass is connected at \( P_{R1} \) and \( P_{R2} \) with \( X_{R1} \) and \( X_{R2} \) of sprung mass. The forces applied at these connecting points are given as the summation of the spring forces that are the products of the lateral deflection and the stiffness of the compliances and the damping forces that are products of lateral deflection speed and the damping co-efficient of the compliances. The forces are given as follows;

\[ F_{yF1} = -(y_{PF1} K_{F1} + \dot{y}_{PF1} : D_{F1}) \]

(7-50)

Substituting Eq.(7-11) to Eq.(7-50);

\[ F_{yF1} = -(y_{F} + l_{F1} \dot{\delta}_{CF}) K_{F1} + (\dot{y}_F + l_{F1} \dot{\delta}_{CF}) D_{F1} \]

(7-50-1)
6. THE DEPLOY OF THE FUNDAMENTAL EQUATIONS

6-1. Equation of sprung mass for lateral motion in the y-axis direction

Submitting Eq.(7-50-1),(7-51-1),(7-52-1),(7-53-1) and (7-54-1) to Eq.(7-1);

\[
M \ddot{v}_y + (v_x \beta + v_x \dot{\beta}) = \begin{cases} 
-y_x + l_x \delta_x, & y_x \in \{y_x + l_x \delta_x, \delta_x \}, \\
-(y_x + l_x \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(y_x + l_x \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(y_x + l_x \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
\end{cases}
\]  

Putting \( v_y = v_x \beta \), then consolidating Eq.(7-1-1) regarding \( \beta, \delta_x, \delta_y, y_x, y_x, y_y \), and \( \delta_y \);  

\[
M \ddot{v}_y + (v_x \beta + v_x \dot{\beta}) = \begin{cases} 
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x, \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
\end{cases}
\]  

Consolidating Eq.(7-1-2);  

\[
M \ddot{v}_y + (v_x \beta + v_x \dot{\beta}) = \begin{cases} 
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x, \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
\end{cases}
\]  

6-2. Equation of sprung mass for rotational motion around the z-axis

Submitting Eq.(7-50-1),(7-51-1),(7-52-1),(7-53-1) and (7-54-1) to Eq.(7-2);  

\[
I \ddot{\phi} + (v_x \beta + v_x \dot{\beta}) = \begin{cases} 
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x, \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
\end{cases}
\]  

Consolidating Eq.(7-2-2) regarding \( \beta, \delta_x, \delta_y, y_x, y_y, \) and \( \delta_y \);  

\[
I \ddot{\phi} + (v_x \beta + v_x \dot{\beta}) = \begin{cases} 
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x, \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
\end{cases}
\]  

Consolidating Eq.(7-2-2);  

\[
I \ddot{\phi} + (v_x \beta + v_x \dot{\beta}) = \begin{cases} 
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x, \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
-(l_Fl_D + l_f + l_F^2 + \delta_x), & y_x \in \{y_x + l_x \delta_x \}, \\
\end{cases}
\]  

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6-3. Equation of front un-sprung mass for lateral motion in the y-axis direction

Submitting Eq.(7-15),(7-16),(7-50-1) and (7-51-1) to Eq.(7-3);

\[
M_F \left( \dddot{y}_F + \dot{y}_F + x_F \dot{\phi} + v_x \dot{\phi} + v_s \delta_{CF} \right) \\
+ \left( \dot{y}_F + l_{F1} \delta_{CF} \right) k_{F1} + \left( \dot{y}_F + l_{F1} \delta_{CF} \right) D_{CF1} \cos \delta_{CF} \\
+ \left( \ddot{y}_F + l_{F2} \delta_{CF} \right) k_{F2} + \left( \dot{y}_F + l_{F2} \delta_{CF} \right) D_{CF2} \cos \delta_{CF} \\
+ F_{ys} = 0
\]  
(7-3-1)

Consolidating Eq.(7-3-1) regarding \( \phi, \beta, \delta_{CF}, y_F \) and \( \delta_s \) putting \( \cos \delta_{CF} \approx 1, v_y = \beta_{yX} \);

\[
M_F x_F \dddot{\phi} \\
+ M_F v_x \dot{\phi} \\
+ M_F v_x \dot{\beta} \\
+ \left( M_F v_x + l_{F1} D_{F1} + l_{F2} D_{F2} \right) \dot{\delta}_{CF} \\
+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) \delta_{CF} \\
+ M_F \dddot{y}_F \\
+ \left( D_{F1} + D_{F2} \right) \ddot{y}_F \\
+ \left( k_{F1} + k_{F2} \right) \dot{y}_F \\
+ F_{ys} = 0
\]  
(7-3-2)

Substituting \( F_{ys} \) of Eq.(7-7) to \( F_{ys} \) of (7-3-2);

\[
M_F x_F \dddot{\phi} \\
+ M_F v_x \dot{\phi} \\
+ M_F v_x \dot{\beta} \\
+ \left( M_F v_x + l_{F1} D_{F1} + l_{F2} D_{F2} \right) \dot{\delta}_{CF} \\
+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) \delta_{CF} \\
+ M_F \dddot{y}_F \\
+ \left( D_{F1} + D_{F2} \right) \ddot{y}_F \\
+ \left( k_{F1} + k_{F2} \right) \dot{y}_F \\
+ \left[ M_S \left( \dot{v}_{y3} + v_{y3} \dot{\phi}_3 \right) - F_{yst} \cos \left( \delta_s + \delta_{CF} \right) + SF_F \right] / \cos \delta_s = 0
\]  
(7-3-3)

Substituting \( SF_F \) of Eq.(7-30) to Eq.(7-3-3);

\[
M_F x_F \dddot{\phi} \\
+ M_F v_x \dot{\phi} \\
+ M_F v_x \dot{\beta} \\
+ \left( M_F v_x + l_{F1} D_{F1} + l_{F2} D_{F2} \right) \dot{\delta}_{CF} \\
+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) \delta_{CF} \\
+ M_F \dddot{y}_F \\
+ \left( D_{F1} + D_{F2} \right) \ddot{y}_F \\
+ \left( k_{F1} + k_{F2} \right) \dot{y}_F \\
+ \left[ M_S \left( \dot{v}_{y3} + v_{y3} \dot{\phi}_3 \right) - F_{yst} \cos \left( \delta_s + \delta_{CF} \right) \right] / \cos \delta_s = 0
\]  
(7-3-4)

Substituting Eq.(7-27) to \( v_{y3} \), Eq.(7-18) to \( \delta_s \) and Eq.(7-51-1) to \( F_{yst} \) of Eq.(7-3-4);

\[
M_F x_F \dddot{\phi} \\
+ M_F v_x \dot{\phi} \\
+ M_F v_x \dot{\beta} \\
+ \left( M_F v_x + l_{F1} D_{F1} + l_{F2} D_{F2} \right) \dot{\delta}_{CF} \\
+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) \delta_{CF} \\
+ M_F \dddot{y}_F \\
+ \left( D_{F1} + D_{F2} \right) \ddot{y}_F \\
+ \left( k_{F1} + k_{F2} \right) \dot{y}_F \\
+ \left[ M_S \left( \dot{v}_{y3} + v_{y3} \dot{\phi}_3 \right) - F_{yst} \cos \left( \delta_s + \delta_{CF} \right) \right] / \cos \delta_s = 0
\]  
(7-3-5)

Consolidating Eq.(7-3-5) putting \( v_{y3} \approx v_{xX}, \cos \delta_s \approx 1, \)
\( \cos (\delta_{CF} + \delta_{CF}) \approx 1 \), then putting \( v_{y} = v_{x} \beta ; \) that is,
\( \dddot{v}_{y} = \dddot{v}_{x} \beta + \dot{v}_{x} \dot{\beta} \);

\[
(M_F + M_S) x_F \dddot{\phi} \\
+ \left( (M_F + M_S) \dot{v}_x + (x_F + l_{F1}) FC_{F_v} \right) \dot{\phi} \\
+ (M_F + M_S) \dot{v}_x \dot{\beta} \\
+ FC_{F_v} \beta \\
+ M_S l_{F2} \delta_{CF} \\
+ \left( (M_F + M_S) \dot{v}_x + l_{F1} D_{F1} + l_{F2} D_{F2} + l_{F3} D_{F3} + FC_{F_v} (l_{F2} + l_{F3}) / v_y \right) \dot{\delta}_{CF} \\
+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} + l_{F3} k_{F3} \right) \delta_{CF} \\
+ (M_F + M_S) \dot{v}_x \\
+ \left( D_{F1} + D_{F2} + D_{F3} + FC_{F_v} / v_y \right) \ddot{y}_F \\
+ \left( k_{F1} + k_{F2} + k_{F3} \right) \dot{y}_F \\
+ M_S l_{F2} \delta_{s} \\
+ \left( M_S v_y + l_{F2} D_{F2} + FC_{F_v} (l_{F3} + l_{F2}) / v_y \right) \dot{\delta}_{s} \\
+ l_{F1} k_{F1} \delta_{s} \\
= -D_{F2} \dot{y}_y - k_{F1} v_y \
\]  
(7-3-6)
6-4. Equation of front un-sprung mass for rotational motion around the $z$-axis

Substituting Eq.(7-16),(7-50-1),(7-51-1) and (7-32) to Eq.(7-4):

$$I_F \left( \dot{\phi} + \delta_{CF} \right)$$
$$+ I_F \left( \left( y_F + l_{F1} \delta_{CF} \right) k_{F1} + \left( y_F + l_{F1} \delta_{CF} \right) D_{F1} \right) \cos \delta_{CF}$$
$$+ l_{F2} \left( \left( y_F + l_{F2} \delta_{CF} \right) k_{F2} + \left( y_F + l_{F2} \delta_{CF} \right) D_{F2} \right) \cos \delta_{CF}$$
$$- \left( k_{FS} \delta_S + D_{FS} \delta_S \right)$$
$$+ l_{FS} F_{yS} = 0$$

(7-4-1)

Consolidating Eq.(7-4-1) putting $\cos \delta_{CF} \approx 1$, $\cos \delta_{CR} \approx 1$;

$$I_F \dot{\phi}$$
$$+ I_F \delta_{CF}$$
$$+ \left( l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1} D_{F1} + l_{F2} D_{F2} \right) y_F$$
$$+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) y_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} F_{yS} = 0$$

(7-4-2)

Substituting $F_y$ of Eq.(7-7) to $F_y$ of Eq.(7-4-2):

$$I_F \dot{\phi}$$
$$+ I_F \delta_{CF}$$
$$+ \left( l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1} D_{F1} + l_{F2} D_{F2} \right) y_F$$
$$+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) y_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} \left( M_{yS} \left( \dot{y}_{zS} + v_{xS} \dot{\phi}_S \right) \right)$$
$$- F_{yS} \cos \left( \delta_S + \delta_{CF} \right)$$
$$+ SF_y \cos \delta_S = 0$$

(7-4-3)

Substituting Eq.(7-30) to $SF_y$ of Eq.(7-4-3):

$$I_F \dot{\phi}$$
$$+ I_F \delta_{CF}$$
$$+ \left( l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1} D_{F1} + l_{F2} D_{F2} \right) y_F$$
$$+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) y_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} \left( M_{yS} \left( \dot{y}_{zS} + v_{xS} \dot{\phi}_S \right) \right)$$
$$- F_{yS} \cos \left( \delta_S + \delta_{CF} \right)$$
$$+ \left( l_{FS} M_{yS} v_s + l_{FS} l_{FS} D_{FS} \right) \dot{\phi}$$
$$+ \left( l_{FS} M_{yS} k_s + l_{FS} l_{FS} D_{FS} \right) \dot{\phi}$$
$$+ \left( l_{FS} D_{FS} + l_{FS} D_{FS} \right) \dot{y}_F$$
$$+ \left( l_{FS} k_s + l_{FS} k_s \right) \dot{y}_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} v_s M_{yS} \dot{\phi}$$
$$+ l_{FS} FC \dot{\phi}$$
$$= -l_{FS} D_{yS} y_{yS} - l_{FS} k_s y_{yS}$$

(7-4-4)

Substituting $F_{yS}$ of Eq.(7-52-1) to $V_{ps}$ and $\phi_{3}$ of Eq.(7-52-1);

$$I_F \dot{\phi}$$
$$+ I_F \delta_{CF}$$
$$+ \left( l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1} D_{F1} + l_{F2} D_{F2} \right) y_F$$
$$+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) y_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} \left( M_{yS} \left( \dot{y}_{zS} + v_{xS} \dot{\phi}_S \right) \right)$$
$$- F_{yS} \cos \left( \delta_S + \delta_{CF} \right)$$
$$+ \left( l_{FS} M_{yS} v_s + l_{FS} l_{FS} D_{FS} \right) \dot{\phi}$$
$$+ \left( l_{FS} M_{yS} k_s + l_{FS} l_{FS} D_{FS} \right) \dot{\phi}$$
$$+ \left( l_{FS} D_{FS} + l_{FS} D_{FS} \right) \dot{y}_F$$
$$+ \left( l_{FS} k_s + l_{FS} k_s \right) \dot{y}_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} v_s M_{yS} \dot{\phi}$$
$$+ l_{FS} FC \dot{\phi}$$

$\approx 1$, $\cos(\delta_S + \delta_{CF}) \approx 1$, $v_y = v_x \dot{\beta}$, That is, $\dot{v}_r = v_x \dot{\beta} + v_x \dot{\beta} \approx v_x \dot{\beta}$;

$$I_F \dot{\phi}$$
$$+ I_F \delta_{CF}$$
$$+ \left( l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} \right) \delta_{CF}$$
$$+ \left( l_{F1} D_{F1} + l_{F2} D_{F2} \right) y_F$$
$$+ \left( l_{F1} k_{F1} + l_{F2} k_{F2} \right) y_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} \left( M_{yS} \left( \dot{y}_{zS} + v_{xS} \dot{\phi}_S \right) \right)$$
$$- F_{yS} \cos \left( \delta_S + \delta_{CF} \right)$$
$$+ \left( l_{FS} M_{yS} v_s + l_{FS} l_{FS} D_{FS} \right) \dot{\phi}$$
$$+ \left( l_{FS} M_{yS} k_s + l_{FS} l_{FS} D_{FS} \right) \dot{\phi}$$
$$+ \left( l_{FS} D_{FS} + l_{FS} D_{FS} \right) \dot{y}_F$$
$$+ \left( l_{FS} k_s + l_{FS} k_s \right) \dot{y}_F$$
$$- D_{FS} \delta_S$$
$$- k_{FS} \delta_S$$
$$+ l_{FS} v_s M_{yS} \dot{\phi}$$
$$+ l_{FS} FC \dot{\phi}$$
$$= -l_{FS} D_{yS} y_{yS} - l_{FS} k_s y_{yS}$$

(7-4-5)
6-5. Equation of rear un-sprung mass for lateral motion in the y-axis direction

Submitting Eq.(7-24),(7-25),(7-53-1),(7-54-1) and (7-31) to Eq.(7-5):

\[
\dot{v}_y + \frac{M_y}{J_x} \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]

Consolidating Eq.(7-5-1) putting \( \cos \delta_{CR} \approx 1, v_y = v_x \beta; \)

\[
M_y \dot{v}_y + \frac{M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR} \]

6-6. Equation of rear un-sprung mass for rotational motion around the z-axis

Submitting Eq.(7-25),(7-53-1),(7-54-1) and (7-31) to Eq.(7-6):

\[
\dot{v}_y + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]

Consolidating Eq.(7-6-1) putting \( v_y = v_x \beta; \)

\[
I_y \ddot{\phi} + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]

6-7. Equation of steering system mass for rotational motion around the z-axis

Substituting the reactive steering force \( F_{ST} \) of Eq.(7-7) to \( F_{ST} \) of Eq.(7-8);

\[
I_y \ddot{\phi} - l_{s1} M_y \phi (v_y, v_x \beta) + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]

Consolidating Eq.(7-8-1); \( I_y \ddot{\phi} = (l_{s1} - l_{s2}) [F_{ST} \cos (\delta_S + \delta_{CR})]
\]

\[
I_y \ddot{\phi} - l_{s1} M_y \phi (v_y, v_x \beta) + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]

Consolidating Eq.(7-8-4) putting \( v_y = v_x \beta, v_z \beta \)

\[
(l_{s1} - l_{s2}) M_y \phi (v_y, v_x \beta) + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]

Consolidating Eq.(7-8-5) putting \( v_y = v_x \beta, v_z \beta \)

\[
(l_{s1} - l_{s2}) M_y \phi (v_y, v_x \beta) + \frac{1}{J_x} \frac{L_y M_y}{J_x} \phi \phi + \frac{1}{J_x} \frac{L_y M_y}{J_x} \delta_{CR} \cos \delta_{CR}
\]
The deployed fundamental equations, that is, Eq.(7-2-2) for $\phi$, Eq.(7-1-3) for $\beta$, Eq.(7-4-6) for $\delta_{CF}$, Eq.(7-3-6) for $y_F$, Eq.(7-6-2) for $CR$, Eq.(7-5-2) for $y_R$ and Eq.(7-8-5) for $s_\delta$, are described with regard to the matrix as shown in Eq.(7-55);

\[
\begin{bmatrix}
 c_{11}D^2 & 0 & c_{12}D + c_{20} & c_{13}D + c_{30} & c_{14}D + c_{40} & c_{15}D + c_{50} & c_{16}D + c_{60}
 d_{11}D & d_{12}D & d_{13}D + d_{30} & d_{14}D + d_{40} & 0 & d_{15}D + d_{50} & d_{16}D + d_{60}
 e_{12}D^2 & 0 & e_{13}D^2 + e_{30}D + e_{30} & e_{14}D^2 + e_{40} & 0 & e_{16}D^2 + e_{60} & 0
 f_{12}D^2 + f_{22}D & f_{13}D & f_{23}D + f_{30} & f_{14}D^2 + f_{40}D + f_{40} & 0 & f_{24}D + f_{40} & 0
 g_{12}D^2 + g_{22}D & g_{13}D & g_{23}D + g_{30} & g_{14}D^2 + g_{40}D + g_{40} & 0 & g_{24}D + g_{40} & 0
 h_{12}D^2 + h_{22}D & h_{13}D & h_{23}D + h_{30} & h_{14}D^2 + h_{40}D + h_{40} & 0 & h_{24}D + h_{40} & 0
 s_{11}D + s_{10} & s_{12}D + s_{20} & s_{13}D + s_{30} & s_{14}D^2 + s_{40} & 0 & s_{16}D^2 + s_{60} & 0
\end{bmatrix}
\begin{bmatrix}
 \phi \\
 \beta \\
 \delta_{CF} \\
 y_F \\
 y_R \\
 \delta_{\delta} \\
 s_\delta + s_{\delta 0}
\end{bmatrix} = \begin{bmatrix}
 c_{11}D + c_{10} \\
 d_{11}D + d_{10} \\
 e_{12}D^2 + e_{10}D + e_{10} \\
 f_{12}D^2 + f_{10}D + f_{10} \\
 g_{12}D^2 + g_{10}D + g_{10} \\
 h_{12}D^2 + h_{10}D + h_{10} \\
 s_{11}D + s_{10} + s_{\delta 10}
\end{bmatrix}
\]

(7-55)

Where, \(D\) is the differential calculus operator. The contents of \(c_{ij}, \ldots, s_{80}\) are shown as follows;

\[
\begin{align*}
c_{11} &= I \\
c_{31} &= -(x_{F1}l_{F1}D_{F1} + x_{F2}l_{F2}D_{F2} + x_{S}l_{FS}D_S) \\
c_{30} &= -(x_{F1}l_{F1}k_{F1} + x_{F2}l_{F2}k_{F2} + x_{S}l_{FS}k_S) \\
c_{41} &= -(x_{F1}l_{F1}D_{F1} + x_{F2}l_{F2}D_{F2} + x_{S}l_{FS}D_S) \\
c_{40} &= -(x_{F1}k_{F1} + x_{F2}k_{F2} + x_{S}k_S) \\
c_{51} &= -(x_{R1}l_{R1}D_{R1} + x_{R2}l_{R2}D_{R2}) \\
c_{50} &= -(x_{R1}l_{R1}k_{R1} + x_{R2}l_{R2}k_{R2}) \\
c_{61} &= -(x_{R1}D_{R1} + x_{R2}D_{R2}) \\
c_{60} &= -(x_{R1}k_{R1} + x_{R2}k_{R2}) \\
c_{71} &= -x_{S}l_{S2}D_S \\
c_{70} &= -x_{S}l_{S2}k_S \\
c_{81} &= +x_{S}D_S \\
c_{80} &= +x_{S}k_S \\
d_{10} &= Mv_x \\
d_{21} &= Mv_x \\
d_{31} &= -(l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S) \\
d_{30} &= -(l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S) \\
d_{41} &= -(D_{F1} + D_{F2} + D_S) \\
d_{40} &= -(k_{F1} + k_{F2} + k_S) \\
d_{51} &= -(l_{R1}D_{R1} + l_{R2}D_{R2}) \\
d_{50} &= -(l_{R1}k_{R1} + l_{R2}k_{R2}) \\
d_{61} &= -(D_{R1} + D_{R2}) \\
d_{60} &= -(k_{R1} + k_{R2}) \\
d_{71} &= -l_{S2}D_S \\
d_{70} &= -l_{S2}k_S \\
d_{81} &= +D_S \\
d_{80} &= +k_S
\end{align*}
\]

(7-55-1) (7-55-2)
\[ e_{11} = I_x + l_{rs} x_p M_s \]
\[ e_{10} = I_{rs} y_p M_s + l_{rs} (x_F + l_{rt}) F_{C_p} / v_s \]
\[ e_{21} = I_{rs} y_p M_s \]
\[ e_{20} = I_{rs} F_{C_p} \]
\[ e_{31} = I_x + l_{rs} / l_{rs} M_s \]
\[ e_{30} = l_{rs} / l_{rs} M_s \]
\[ e_{41} = l_{rs} / l_{rs} M_s \]
\[ e_{40} = I_x + l_{rs} / l_{rs} M_s \]
\[ e_{51} = I_x + l_{rs} / l_{rs} M_s \]
\[ e_{50} = I_x + l_{rs} / l_{rs} M_s \]
\[ e_{61} = +D_{r1} + D_{r2} + RC_p / v_x \]
\[ e_{60} = +k_{ri} + k_{r2} \]

\[ s_{11} = +l_s + l_{rs} x_F M_s \]
\[ s_{10} = -l_{s1} M_s + x_F + l_{rt} F_{C_p} / v_s \]
\[ s_{21} = -l_{s1} M_s \]
\[ s_{20} = +l_{rt} F_{C_p} \]
\[ s_{31} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{30} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{41} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{40} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{51} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{50} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{61} = +D_{r1} + D_{r2} + RC_p / v_x \]
\[ s_{60} = +k_{ri} + k_{r2} \]

\[ h_{11} = x_R M_s \]
\[ h_{10} = M_R y_s + (x_R + l_{rt}) F_{C_p} / v_s \]
\[ h_{21} = M_R y_s \]
\[ h_{20} = +RC_p \]
\[ h_{31} = M_R y_s + l_{r1} D_{r1} + l_{r2} D_{r2} + l_{rt} RC_p / v_x \]
\[ h_{50} = +l_{ri} k_{ri} + l_{r2} k_{r2} \]

\[ s_{11} = +l_s + l_{rs} x_F M_s \]
\[ s_{10} = -l_{s1} M_s + x_F + l_{rt} F_{C_p} / v_s \]
\[ s_{21} = -l_{s1} M_s \]
\[ s_{20} = +l_{rt} F_{C_p} \]
\[ s_{31} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{30} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{41} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{40} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{51} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{50} = +l_s + l_{rs} / l_{rs} M_s \]
\[ s_{61} = +D_{r1} + D_{r2} + RC_p / v_x \]
\[ s_{60} = +k_{ri} + k_{r2} \]

\[ g_{11} = I_R \]
\[ g_{10} = +l_{rt} (x_R + l_{rt}) F_{C_p} / v_s \]
\[ g_{20} = +l_{rt} RC_p \]
\[ g_{31} = +l_r D_{r1} + l_R D_{r2} \]
\[ g_{30} = +l_r / l_r M_s \]
\[ g_{41} = +l_r D_{r1} + l_R D_{r2} + l_{rt} RC_p / v_x \]
\[ g_{40} = +l_r / l_r M_s \]

\[ (7-55-3) \]

\[ f_{11} = +(M_x + M_s) x_F \]
\[ f_{10} = +M_x + M_s x_F + (x_F + l_{rt}) F_{C_p} / v_s \]
\[ f_{21} = +M_x + M_s x_F \]
\[ f_{20} = +F_{C_p} \]
\[ f_{31} = +M_x + M_s x_F \]
\[ f_{30} = +M_x + M_s x_F + l_{r1} D_{r1} + l_{r2} D_{r2} + l_{rs} D_s + (l_{s1} + l_{rt}) F_{C_p} / v_s \]
\[ f_{40} = k_{f1} + k_{f2} + k_s \]
\[ f_{50} = +l_{s2} M_s \]
\[ f_{60} = +l_{s2} M_s \]

\[ (7-55-4) \]

\[ g_{11} = I_R \]
\[ g_{10} = +l_{rt} (x_R + l_{rt}) F_{C_p} / v_s \]
\[ g_{20} = +l_{rt} RC_p \]
\[ g_{31} = +l_r D_{r1} + l_R D_{r2} \]
\[ g_{30} = +l_r / l_r M_s \]
\[ g_{41} = +l_r D_{r1} + l_R D_{r2} + l_{rt} RC_p / v_x \]
\[ g_{40} = +l_r / l_r M_s \]

\[ (7-55-5) \]

\[ (7-55-6) \]

\[ (7-55-7) \]
SYMBOLS;

<CONSTANTS>

\[ M \] : Sprung mass (kg)

\[ M_F \] : Un-sprung mass of front suspension (kg)

\[ M_R \] : Un-sprung mass of rear suspension (kg)

\[ M_S \] : Un-sprung mass of steering system around the king pin (kg)

\[ J \] : Yawing moment of inertia of sprung mass (kg-m^2)

\[ J_F \] : Yawing moment of inertia of un-sprung mass of front suspension (kg-m^2)

\[ J_R \] : Yawing moment of inertia of un-sprung mass of rear suspension (kg-m^2)

\[ J_S \] : Moment of inertia of steering system around the king pin (kg-m^2)

\[ x_F \] : x value of the location of the front suspension gravity center (m)

\[ x_{F1} \] : x value of the location of front connecting point of front suspension (m)

\[ x_{F2} \] : x value of the location of rear connecting point of front suspension (m)

\[ x_S \] : x value of the location of connecting point of steering system (m)

\[ x_R \] : x value of the location of the rear suspension gravity center (m)

\[ x_{R1} \] : x value of the location of front connecting point of rear suspension (m)

\[ x_{R2} \] : x value of the location of rear connecting point of rear suspension (m)

\[ l_{FS} \] : x1 value of the location of king pin (m)

\[ l_{S1} \] : x3 value of the location of connecting point of steering system gravity center (m)

\[ l_{S2} \] : x3 value of the location of connecting point of steering linkage (m)

\[ l_{FT} \] : x3 value of the location of applying point of front side force (m)

\[ l_{R1} \] : x3 value of the location of front connecting point from rear suspension gravity center (m)

\[ l_{R2} \] : x3 value of the location of rear connecting point from rear suspension gravity center (m)

\[ l_{R3} \] : x3 value of the location of applying point of rear side force (m)

\[ FC_{pa} \] : Front cornering power(total of left and right)(N/rad)

\[ RC_{pa} \] : Rear cornering power(total of left and right)(N/rad)

\[ k_{F1} \] : Lateral stiffness of front connecting point of front suspension(N/m)

\[ k_{F2} \] : Lateral stiffness of rear connecting point of front suspension(N/m)

\[ k_{R1} \] : Lateral stiffness of front connecting point of rear suspension(N/m)

\[ k_{R2} \] : Lateral stiffness of rear connecting point of rear suspension(N/m)

\[ k_S \] : Lateral stiffness of steering linkage(N/m)

\[ D_{F1} \] : Damping coefficient of front connecting point of front suspension(N/m-s)

\[ D_{F2} \] : Damping coefficient of rear connecting point of front suspension(N/m-s)

\[ D_{R1} \] : Damping coefficient of front connecting point of rear suspension(N/m-s)

\[ D_{R2} \] : Damping coefficient of rear connecting point of rear suspension(N/m-s)

\[ D_s \] : Damping coefficient of steering system(N/m-s)
<VARIABLES>

\(v_x\) : x direction velocity of sprung mass (m/s)

\(v_y\) : y direction velocity of sprung mass (m/s)

\(v_{x_1}\) : x direction velocity of un-sprung mass of front suspension (m/s)

\(v_{y_1}\) : y direction velocity of un-sprung mass of front suspension (m/s)

\(v_{x_2}\) : x direction velocity of un-sprung mass of rear suspension (m/s)

\(v_{y_2}\) : y direction velocity of un-sprung mass of rear suspension (m/s)

\(v_{x_3}\) : x direction velocity of un-sprung mass of steering (m/s)

\(v_{y_3}\) : y direction velocity of un-sprung mass of steering (m/s)

\(\beta\) : Side slip angle of gravity center of sprung mass (rad)

\(\phi\) : Yawing angle at gravity center of sprung mass (rad)

\(\beta_1\) : Side slip angle of gravity center of un-sprung mass of front suspension (rad)

\(\phi_1\) : Yawing angle at gravity center of un-sprung mass of front suspension (rad)

\(\beta_2\) : Side slip angle of gravity center of un-sprung mass of rear suspension (rad)

\(\phi_2\) : Yawing angle at gravity center of un-sprung mass of rear suspension (rad)

\(\beta_3\) : Side slip angle of gravity center of un-sprung mass of steering system (rad)

\(\phi_3\) : Yawing angle at gravity center of un-sprung mass of steering system (rad)

\(\delta_{CF}\) : Compliance steer angle of un-sprung mass of front suspension (rad)

\(\delta_{CR}\) : Compliance steer angle of un-sprung mass of rear suspension (rad)

\(\delta_s\) : Compliance steer angle of un-sprung mass of steering system to the front suspension (rad)

\(y_F\) : Lateral compliance of front suspension (m)

\(y_R\) : Lateral compliance of rear suspension (m)

\(F_{yF1}\) : Lateral force between P_{F1} of front suspension and Q_{F1} of sprung mass (N)

\(F_{yF2}\) : Lateral force between P_{F2} of front suspension and Q_{F2} of sprung mass (N)

\(F_{yR1}\) : Lateral force between P_{R1} of rear suspension and Q_{R1} of sprung mass (N)

\(F_{yR2}\) : Lateral force between P_{R2} of rear suspension and Q_{R2} of sprung mass (N)

\(F_{yST}\) : Restoring force between P_{ST} of steering system and Q_{ST} of sprung mass (N)

\(T_{RS}\) : Restoring torque around the king pin center, P_{RS} (N)

\(S_{yF}\) : Front side force (Total of left and right) (N)

\(S_{yR}\) : Rear side force (Total of left and right) (N)

\(D_{yF}\) : Front drag force (Total of left and right) (N)

\(D_{yR}\) : Rear drag force (Total of left and right) (N)

\(C_{yF}\) : Front cornering force (Total of left and right) (N)

\(C_{yR}\) : Rear cornering force (Total of left and right) (N)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Fig.4&amp;6</th>
<th>Fig.5</th>
<th>Symbol</th>
<th>Fig.4</th>
<th>Fig.6</th>
<th>Fig.5</th>
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<td>$FC_p$ (N/rad)</td>
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