
7 DOF Vehicle Model for Understanding Vehicle Fluctuation During Straight Running

Fujio Momiyama
HORIKIRI, INC.

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ABSTRACT

This paper concerns an introduction on how to lead the 7 degrees of freedom (7DOF) model for understanding the vehicle-fluctuation during straight running. Despite the steering wheel being fixed at on-center by anchor rod connected to the floor, vehicle run with a very slow lateral fluctuation, of which amplitude is small but becomes maximum at a certain speed zone, was observed by a vehicle test on a flat road of the proving ground. For understanding this phenomenon, I provided the motion equation of the 7DOF model. By analysis using the motion equation, the existence of a speed zone where the amplitude becomes maximum is confirmed. This appears when the rear compliance steer is the proverse compliance steer and disappears by changing it to the adverse compliance steer. Where, the adverse/proverse compliance steer mean the characteristics of compliance steer that increase/decrease the side-slip angle versus the vector direction of the vehicle⁽¹⁾.

INTRODUCTION

I have explained the existence of fluctuation during straight running caused by the compliance of steering/suspension system in SAE 2001-01-3433⁽¹⁾ and confirmed the existence of a very slow yaw-peak of which frequency changes with vehicle speed by a vehicle test conforming to ISO/TR8726 in IMeCE2001/DE-23259⁽²⁾. The slow yawing coming from the angular velocity change of steering and front suspension was found by an eigen vector analysis using the 7DOF model analysis. This paper describes the leading process of the 7DOF model equation whose description had been abridged because of the paper space limitation, and the existence of the speed zone where amplitude becomes maximum that appears when the rear compliance steer is the proverse compliance steer and disappears by changing it to the adverse compliance steer.

REVIEW OF SAE 2000-01-3433 & IMECE2001/DE-23259

A very low frequency vehicle fluctuation of which amplitude becomes maximum at 60 or 80 km/h is confirmed as shown in Fig.1⁽¹⁾. The test was done by a large sized bus fixing the steering wheel on center with anchor rod for avoiding the influence of steering wheel input. Fig.2⁽²⁾ shows the yaw-gain tested in conformance with ISO/TR8726. The heave (low peak) is observed at the frequency zone from 0.1 to 0.3Hz. This low frequency corresponds to that of Fig.1. In order to understand this low frequency and its speed-dependence characteristics, I provided the 7DOF vehicle model in considering the lateral and yawing motions of the sprung mass, front un-sprung mass, rear un-sprung mass and compliance at steering link, front suspension links and rear suspension links as shown in Fig.3⁽²⁾

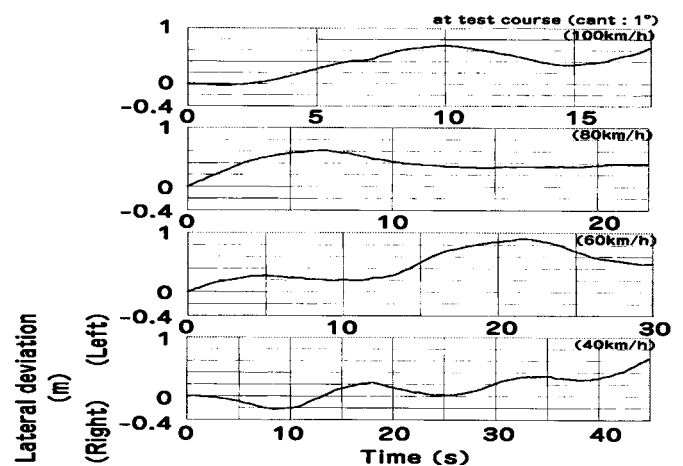


Fig.1 Vehicle fluctuation during straight running

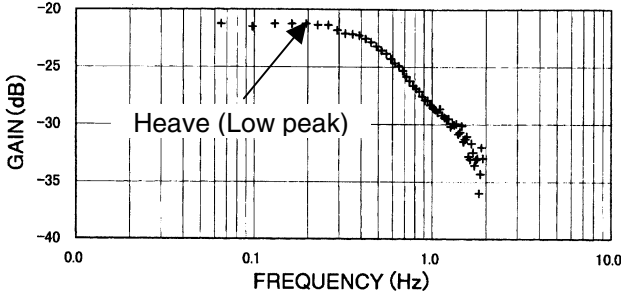


Fig.2 Heave/Gain-Peak at the frequency zone from 0.1 Hz to 0.3Hz

M_S/M_R is moving at the speed V_S/V_R with the side slip angle, β_F / β_R .

The lateral and rotational motion equations of sprung-mass are written as follows;

$$M(\dot{v}_y + v_x \dot{\phi}) = -F_{yF1} - F_{yF2} - F_{yR1} - F_{yR2} - F_{yST} \quad (7-1)$$

$$M(\dot{v}_y + v_x \dot{\phi}) + F_{yF1} + F_{yF2} + F_{yR1} + F_{yR2} + F_{yST} = 0$$

$$I\ddot{\phi} = -x_{F1}F_{yF1} - x_{F2}F_{yF2} - x_S F_{yST} - x_{R1}F_{yR1} - x_{R2}F_{yR2} \quad (7-2)$$

$$I\ddot{\phi} + x_{F1}F_{yF1} + x_{F2}F_{yF2} + x_S F_{yST} + x_{R1}F_{yR1} + x_{R2}F_{yR2} = 0$$

The lateral and rotational motion equations of front sprung-mass are written as follows;

$$M_F(\dot{v}_{y1} + v_{x1}\dot{\phi}_1) = +F_{yF1} \cos \delta_{CF} + F_{yF2} \cos \delta_{CF} - F_{yS} \quad (7-3)$$

$$M_F(\dot{v}_{y1} + v_{x1}\dot{\phi}_1) - F_{yF1} \cos \delta_{CF} - F_{yF2} \cos \delta_{CF} + F_{yS} = 0$$

$$I_F\ddot{\phi}_1 = l_{F1}F_{yF1} \cos \delta_{CF} + l_{F2}F_{yF2} \cos \delta_{CF} - l_{FS}F_{yS} + T_{FS} \quad (7-4)$$

$$I_F\ddot{\phi}_1 - l_{F1}F_{yF1} \cos \delta_{CF} - l_{F2}F_{yF2} \cos \delta_{CF} + l_{FS}F_{yS} - T_{FS} = 0$$

The lateral and rotational motion equations of rear sprung-mass are written as follows;

$$M_R(\dot{v}_{y2} + v_{x2}\dot{\phi}_2) = +F_{yR1} \cos \delta_{CR} + F_{yR2} \cos \delta_{CR} - SF_R \quad (7-5)$$

$$M_R(\dot{v}_{y2} + v_{x2}\dot{\phi}_2) - F_{yR1} \cos \delta_{CR} - F_{yR2} \cos \delta_{CR} + SF_R = 0$$

$$I_R\ddot{\phi}_2 = l_{R1} \cdot F_{yR1} + l_{R2} \cdot F_{yR2} - l_{RT}SF_R \quad (7-6)$$

$$I_R\ddot{\phi}_2 - l_{R1} \cdot F_{yR1} - l_{R2} \cdot F_{yR2} + l_{RT}SF_R = 0$$

The lateral and rotational motion equations of steering system mass are written as follows;

$$M_S(\dot{v}_{y3} + v_{x3}\dot{\phi}_3) = +F_{yS} \cos \delta_S + F_{yST} \cos(\delta_S + \delta_{CF}) - SF_F \quad (7-7)$$

$$M_S(\dot{v}_{y3} + v_{x3}\dot{\phi}_3) - F_{yS} \cos \delta_S - F_{yST} \cos(\delta_S + \delta_{CF}) + SF_F = 0$$

$$I_S\ddot{\phi}_3 = l_{S1}F_{yS} \cos \delta_S + l_{S2}F_{yST} \cos(\delta_S + \delta_{CF}) - l_{FT}SF_F - T_{FS} \quad (7-8)$$

$$I_S\ddot{\phi}_3 - l_{S1}F_{yS} \cos \delta_S - l_{S2}F_{yST} \cos(\delta_S + \delta_{CF}) + l_{FT}SF_F + T_{FS} = 0$$

Deploying the above equations, (7-1),(7-2),.....,(7-7),(7-8), the matrix,(7-55), is given as shown in the Appendix.

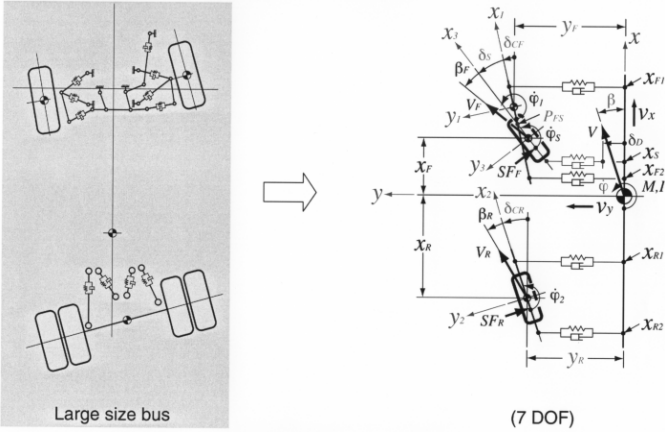


Fig.3 Vehicle Model

MOTION EQUATION OF 7DOF MODEL

In Fig.3, the $x-y$ is the coordinate of the sprung mass, the mass of which is M and the inertia is I , moving at the speed V with the side slip angle β , and yawing velocity $\dot{\phi}$.

The x_1-y_1 is the coordinate of the front un-sprung mass, the mass of which is M_F and the inertia is I_F , deviating y_F and twisting δ_{CF} to x -axis caused by compliance spring/damping elements, K_{F1}/D_{F1} and K_{F2}/D_{F2} which exist in the suspension linkage connecting to the sprung mass, M . The x_3-y_3 is the coordinate of the steering system mass, the mass of which is M_S and the inertia is I_S , connected by king-pin at P_{FS} to the x_1 axis, and steered δ_S by the steering link connected to the sprung mass via compliance spring/damping element, K_S/D_S . The x_2-y_2 is the coordinate of the rear un-sprung mass, the mass of which is M_R and the inertia is I_R , deviating y_R and twisting δ_{CR} to x -axis caused by compliance spring/damping elements, K_{R1}/D_{R1} and K_{R2}/D_{R2} which exist in the suspension linkage connecting to the sprung mass, M .

YAW-GAIN PEAK CONTROL STUDY BY 7DOF MODEL

Using the 7DOF model, parametric studies have been carried out. Fig.4 shows a calculated result in the case of the proverse compliance steer for the front and the proverse compliance steer for the rear. The specification is shown in Table 1 of the Appendix. There are two peaks. One is at 0.3 or 0.4Hz and the other is at 2 or 3 Hz. It is considered that one comes from the natural frequency of sprung mass and the other comes from that of un-sprung mass. The numbers, 40, 60, 80, 100 show the vehicle speed in km/h. It is observed that the yaw-gain becomes maximum at 80 km/h in this case. Fig.5 shows a calculated result in the case of the proverse compliance steer for the front and the “adverse compliance steer for the rear”. Also, the specification is shown in Table 1 of the Appendix. The yaw-gain peak at 80 km/h has become lower and the peak height order has changed from 80, 100, 60, 40 km/h to 40, 60, 80, 100 km/h. Fig.6 shows, as a reference, a case where damping factors are added to the steering system and front suspension system of the case of Fig.4. In this case, the yaw gain peak order becomes 100, 80, 60, 40 km/h.

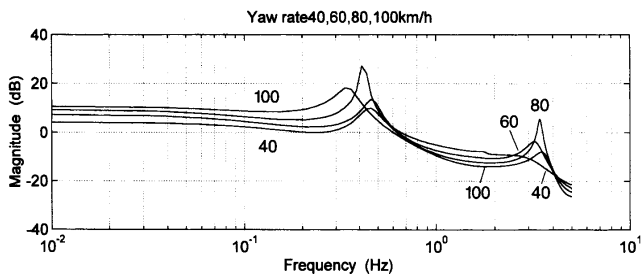


Fig.4 Yaw-gain of front proverse / Rear proverse combination

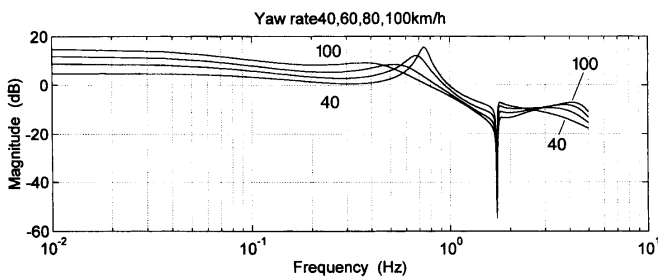


Fig.5 Yaw-gain of front proverse / Rear adverse combination

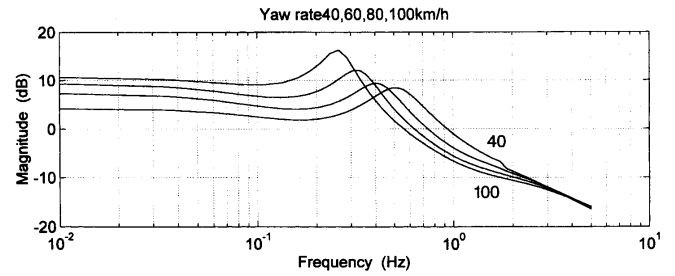


Fig.6 A case where damping factors are added to the steering/front suspension system of the front proverse/Rear proverse combination of Fig.4

DISCUSSION

As the 7DOF model has been subjected to linearization boldly for finding the principle of fluctuation during straight running, we should understand that only the tendency and not the absolute value should be discussed when we use this 7DOF model. Based upon this understanding, the following discussion can be made. The existence of the speed zone where the amplitude becomes maximum is also confirmed by calculation as in Fig.4. This is back-up data to the experiment result shown in Fig.1. By changing the rear compliance steer from proverse to adverse, the tendency has changed as shown in Fig.5. This tendency appears more stable at higher speed zone than lower speed zone. The peak at 3 or 4 Hz in Fig.4 disappeared and then a cave appeared at 1 or 2 Hz zone. Judging from the eigen value analysis (Fig.7) and the eigen vector analysis (Fig.8) done in the previous paper⁽²⁾, the 1 or 2 Hz corresponds to the root C that comes from the rear compliance and the 3 or 4 Hz corresponds to the root D that comes from the front compliance. Therefore the cave is considered to have come from the adverse compliance steer of rear suspension. By adding damping to the front suspension, the peak at 80 km/h in Fig.4 has been lowered as shown in Fig.6. In this case, the tendency shows that stability reduces more at higher speed zone than lower speed zone. This means that Fig.5 and Fig.6 have the opposite tendency as explained in SAE 2001-01-3433⁽¹⁾.

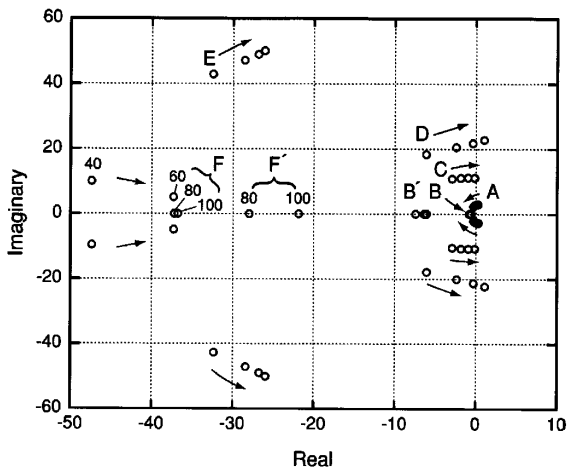


Fig.7 Eigen value analysis of Fig.4 condition

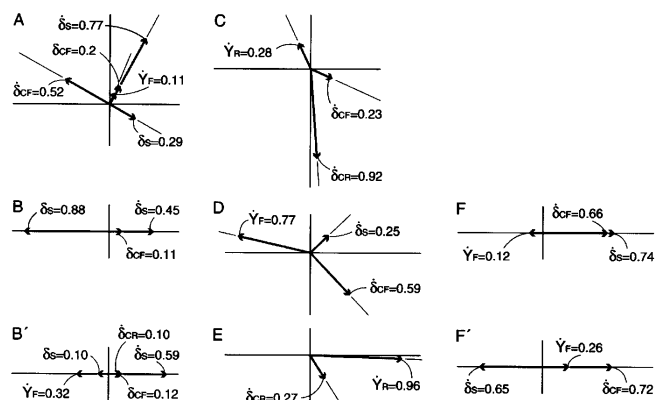


Fig.8 Eigen vector analysis of Fig.4 condition

CONCLUSION

The leading process of 7DOF model is explained and examples of straight running analysis are shown. By constructing such a linear motion equation, it is possible to describe the bode diagram, to do eigen value analysis and eigen vector analysis. This is very useful to understanding the principle. As the existence of the characteristic roots are previously known by eigen value analysis, vehicle dynamics design focusing on the influence of the root becomes possible. By the eigen vector analysis, we can understand the constituents of the phenomenon. Vehicle dynamics design based on the principle is thus possible. The following have become known;

1. It is said generally that the vehicle stability will decrease with an increase of vehicle speed. However, strictly speaking, there are two other cases where decrease at the speed zone and increase occur simply with vehicle speed.

2. The cases due to the combination of front compliance and rear compliance. The case that decreases at a speed zone due to the larger front compliance under the combination of front proverse and rear proverse compliance steer. This disappears by changing the rear compliance from proverse to adverse, or increasing the damping of the steering and front suspension system.

ACKNOWLEDGMENTS

I developed this 7DOF model in 1997 as a part of my doctoral thesis, but did not include it in the final thesis⁽¹⁾ because the vehicle model *considering the unsprung mass effect* did not meet with academic circles. I understood that further discussion would be necessary. I am thankful to SAE for giving me the opportunity to make this presentation, and also wish to thank the persons who show me their interest in my 7DOF model.

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2. Fujio Momiyama, Takashi Sasaki and Hiroyasu Nagae: Confirmation of the Natural Yawing Frequency of Large Sized Vehicle –Distinguishing the Vehicle System from the Driver-Vehicle System-, 2001 ASME International Mechanical Engineering Congress and Exposition, November 11-16, 2001, New York, NY, IMECE2001/DE-23259.

APPENDIX

Here, I describe the continuous process referring to Fig.3, Fig.3-1 in Appendix, on how to deploy the fundamental equations led as (7-1),(7-2),(7-3),(7-4),(7-5),(7-6),(7-7) and (7-8) for getting the matrix, (7-55). Firstly the relation between the sprung-mass coordinate $x-y$, the un-sprung mass coordinates x_1-y_1 and x_2-y_2 , is described, secondly the slip angle of tires is described, thirdly the side forces of tires, fourthly the forces applied between sprung mass and un-sprung mass, and then the process to reach the matrix (7-55) is described.

1. THE RELATION BETWEEN THE SPRUNG-MASS COORDINATE $x-y$ AND THE UN-SPRUNG MASS COORDINATES x_1-y_1

1-1. The relation between front un-sprung mass and sprung mass

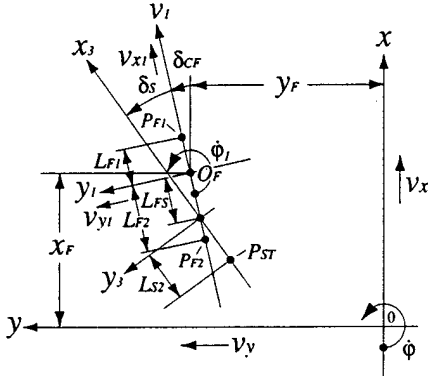


Fig.9 The relation between the coordinate x-y and the coordinates x_1-y_1

General equation between the coordinate x-y and the coordinates x_1-y_1 is written as follows;

$$\begin{cases} x = x_1 \cos \delta_{CF} - y_1 \sin \delta_{CF} + x_F \\ y = y_1 \cos \delta_{CF} + x_1 \sin \delta_{CF} + y_F \end{cases} \quad (7-9,10)$$

They coordinates of P_{F1} , P_{F2} and P_{ST} are shown as follows;

$$y_{PF1} = y_F + l_{F1} \sin \delta_{CF} \approx y_F + l_{F1} \delta_{CF} \quad (7-11)$$

$$y_{PF2} = y_F + l_{F2} \sin \delta_{CF} \approx y_F + l_{F2} \delta_{CF} \quad (7-12)$$

$$y_{PST} = y_F + l_{FS} \sin \delta_{CF} + l_{S2} \sin \delta_S \approx y_F + l_{FS} \delta_{CF} + l_{S2} \delta_S \quad (7-13)$$

The relation between v_y and v_{y1} is shown as follows;

$$\dot{y}_F = v_{y1} \cos \delta_{CF} - v_y - x_F \dot{\phi} \quad (7-14-1)$$

$$y_{y1} = (\dot{y}_F + v_y + x_F \dot{\phi}) / \cos \delta_{CF} \quad (7-14-2)$$

Here, we put $\cos \delta_{CF} \approx 1$;

$$y_{y1} = \dot{y}_F + v_y + x_F \dot{\phi} \quad (7-15)$$

The relation between ϕ , ϕ_1 , δ_{CF} and δ_S is shown as follows;

$$\dot{\phi}_1 = \dot{\phi} + \dot{\delta}_{CF} \quad (7-16)$$

$$\dot{\phi}_3 = \dot{\phi} + \dot{\delta}_{CF} + \dot{\delta}_S \quad (7-18)$$

1-2. The relation between sprung mass and rear un-sprung mass

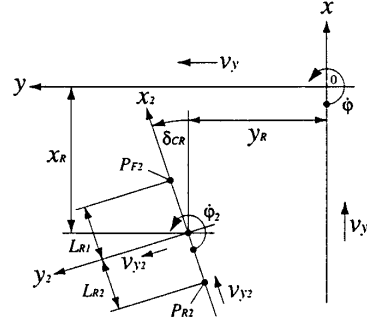


Fig.10 The relation between the coordinate x-y and the coordinates x_2-y_2

General equation between the coordinate x-y and the coordinates x_2-y_2 is written as follows;

$$\begin{cases} x = x_2 \cos \delta_{CR} - y_2 \sin \delta_{CR} + x_R \\ y = y_2 \cos \delta_{CR} + x_2 \sin \delta_{CR} + y_R \end{cases} \quad (7-19,20)$$

They coordinates of P_{R1} and P_{R2} are shown as follows;

$$y_{PF2} = y_R + l_{R1} \sin \delta_{CR} \approx y_R + l_{R1} \cdot \delta_{CR} \quad (7-21)$$

$$y_{PR2} = y_R + l_{R2} \sin \delta_{CR} \approx y_R + l_{R2} \delta_{CR} \quad (7-22)$$

The relation between v_y and v_{y2} is shown as follows;

$$\dot{y}_R = v_{y2} \cos \delta_{CR} - v_y - x_R \dot{\phi} \quad (7-23-1)$$

$$v_{y2} = (\dot{y}_R + v_y + x_R \dot{\phi}) / \cos \delta_{CR} \quad (7-23-2)$$

Here, we put $\cos \delta_{CR} \approx 1$;

$$v_{y2} = \dot{y}_R + v_y + x_R \dot{\phi} \quad (7-24)$$

The relation between ϕ , ϕ_2 and δ_{CR} is shown as follows;

$$\dot{\phi}_2 = \dot{\phi} + \dot{\delta}_{CR} \quad (7-25)$$

2. THE SIDE SLIP ANGLE OF TIRES

2-1. The side slip angle of front tires

The side slip angle of front tire is described as follows;

$$\beta_F = \tan^{-1} \left\{ \frac{v_{y3} + l_{FT} \dot{\phi}_3}{v_{x3}} \right\} \quad (7-26)$$

$$\approx \frac{(v_{y3} + l_{FT} \dot{\phi}_3)}{v_{x3}}$$

Where, the v_{y3} is written as follows from the relation shown in Fig.9;

$$v_{y3} = (\dot{y}_F + v_y + x_F \dot{\phi}) \cos(\delta_{CF} + \delta_S) + l_{FS} \dot{\delta}_{CF} + l_{S2} \dot{\delta}_S \quad (7-27-1)$$

Here, we put $\cos(\delta_{CF} + \delta_S) \approx 1$;

$$v_{y3} = \dot{y}_F + v_y + x_F \dot{\phi} + l_{FS} \dot{\delta}_{CF} + l_{S2} \dot{\delta}_S \quad (7-27-2)$$

Substituting Eq.(7-27-2) to v_{y3} of Eq.(7-26) and Eq.(7-18) to $\dot{\phi}_3$ of Eq.(7-26) and putting $v_{x3} \approx v_x$;

$$\beta_F \approx \left\{ \dot{y}_F + v_y + x_F \dot{\phi} + l_{FS} \dot{\delta}_{CF} + l_{S2} \dot{\delta}_S + l_{FT} \dot{\phi} + l_{FT} \dot{\delta}_{CF} + l_{FT} \dot{\delta}_S \right\} / v_x \quad (7-26-1)$$

Rewriting the Eq.(2-26-1);

$$\beta_F \approx \left\{ \dot{y}_F + v_y + (x_F + l_{FT}) \dot{\phi} + (l_{FS} + l_{FT}) \dot{\delta}_{CF} + (l_{S2} + l_{FT}) \dot{\delta}_S \right\} / v_x \quad (7-26-2)$$

2-2. The side slip angle of rear tire

The side slip angle of rear tire is written as follows;

$$\beta_R = \tan^{-1} \left\{ \frac{v_{y2} + l_{RT} \dot{\phi}_2}{v_{x2}} \right\} \quad (7-28)$$

$$\approx \frac{(v_{y2} + l_{RT} \dot{\phi}_2)}{v_x}$$

Substituting Eq.(7-24) to v_{y2} of Eq.(7-28) and Eq.(7-25) to $\dot{\phi}_2$ of Eq.(7-28) and putting $v_{x2} \approx v_x$;

$$\beta_R = \left\{ \dot{y}_R + v_y + x_R \dot{\phi} + l_{RT} \dot{\phi} + l_{RT} \dot{\delta}_{CR} \right\} / v_x \quad (7-29)$$

$$= \left\{ \dot{y}_R + v_y + (x_R + l_{RT}) \dot{\phi} + l_{RT} \dot{\delta}_{CR} \right\} / v_x$$

3. THE SIDE FORCES OF TIRES

The side force of front tire is described as follows;

$$SF_F \approx FC_P \beta_F$$

$$= FC_P \left\{ \dot{y}_F + v_y + (x_F + l_{FT}) \dot{\phi} + (l_{FS} + l_{FT}) \dot{\delta}_{CF} + (l_{S2} + l_{FT}) \dot{\delta}_S \right\} / v_x \quad (7-30)$$

The side force of rear tire is described as follows;

$$SF_R \approx RC_P \beta_R$$

$$= RC_P \left\{ \dot{y}_R + v_y + (x_R + l_{RT}) \dot{\phi} + l_{RT} \dot{\delta}_{CR} \right\} / v_x \quad (7-31)$$

4. RESTORING TORQUE, T_{FS} AT THE CONNECTING POINT, P_{FS}

Here, we put the torsional spring constant as K_{FS} and the damping coefficient as D_{FS} , then T_{FS} is described as follows;

$$T_{FS} = +K_{FS} \delta_S + D_{FS} \cdot \dot{\delta}_S \quad (7-32)$$

5. THE FORCES APPLIED BETWEEN SPRUNG MASS AND UN-SPRUNG MASS

The front un-sprung mass is connected at P_{F1} and P_{F2} with x_{F1} and x_{F2} of sprung mass and also at P_{ST} with steering force input point x_S . And, the rear un-sprung mass is connected at P_{R1} and P_{R2} with X_{R1} and X_{R2} of sprung mass. The forces applied at these connecting points are given as the summation of the spring forces that are the products of the lateral deflection and the stiffness of the compliances and the damping forces that are products of lateral deflection speed and the damping co-efficient of the compliances. The forces are given as follows;

$$F_{yF1} = -(y_{PF1} K_{F1} + \dot{y}_{PF1} \cdot D_{F1}) \quad (7-50)$$

Substituting Eq.(7-11) to Eq.(7-50);

$$F_{yF1} = -\left\{ (y_F + l_{F1} \delta_{CF}) K_{F1} + (\dot{y}_F + l_{F1} \dot{\delta}_{CF}) D_{F1} \right\} \quad (7-50-1)$$

In the same manner;

$$F_{yF2} = -\left\{ (y_F + l_{F2}\delta_{CF})K_{F2} + (\dot{y}_F + l_{F2}\dot{\delta}_{CF})D_{F2} \right\} \quad (7-51-1)$$

$$F_{yST} = -\left\{ (y_F + l_{FS}\delta_{CF} + l_{S2}\delta_S + y_{DR})K_S + (\dot{y}_F + l_{FS}\dot{\delta}_{CF} + l_{S2}\dot{\delta}_S + \dot{y}_{DR})D_S \right\} \quad (7-52-1)$$

$$F_{yR1} = -\left\{ (y_R + l_{R1}\delta_{CR})K_{R1} + (\dot{y}_R + l_{R1}\dot{\delta}_{CR})D_{R1} \right\} \quad (7-53-1)$$

$$F_{yR2} = -\left\{ (y_R + l_{R2}\delta_{CR})K_{R2} + (\dot{y}_R + l_{R2}\dot{\delta}_{CR})D_{R2} \right\} \quad (7-54-1)$$

6. THE DEPLOY OF THE FUNDAMENTAL EQUATIONS

6-1. Equation of sprung mass for lateral motion in the y-axis direction

Submitting Eq.(7-50-1),(7-51-1),(7-52-1),(7-53-1) and (7-54-1) to Eq.(7-1);

$$\begin{aligned} & M(\dot{y}_y + v_x\dot{\phi}) \\ & - (y_F + l_{F1}\delta_{CF})k_{F1} - (\dot{y}_F + l_{F1}\dot{\delta}_{CF})D_{F1} \\ & - (y_F + l_{F2}\delta_{CF})k_{F2} - (\dot{y}_F + l_{F2}\dot{\delta}_{CF})D_{F2} \\ & - (y_R + l_{R1}\delta_{CR})k_{R1} - (\dot{y}_R + l_{R1}\dot{\delta}_{CR})D_{R1} \\ & - (y_R + l_{R2}\delta_{CR})k_{R2} - (\dot{y}_R + l_{R2}\dot{\delta}_{CR})D_{R2} \\ & - (y_F + l_{FS}\delta_{CF} + l_{S2}\delta_S + y_{DR})k_S - (\dot{y}_F + l_{FS}\dot{\delta}_{CF} + l_{S2}\dot{\delta}_S + \dot{y}_{DR})D_S \\ & = 0 \end{aligned} \quad (7-1-1)$$

Putting $v_y = v_x\beta$, then consolidating Eq.(7-1-1) regarding $\varphi, \beta, \delta_{CF}, \delta_{CR}, y_F, y_R, y_{DR}$ and δ_S ;

$$\begin{aligned} & M(v_x\dot{\beta} + \dot{y}_x\beta + v_x\dot{\phi}) \\ & - (l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S)\dot{\delta}_{CF} \\ & - (l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S)\dot{\delta}_{CF} \\ & - (l_{R1}D_{R1} + l_{R2}D_{R2})\dot{\delta}_{CR} \\ & - (l_{R1}k_{R1} + l_{R2}k_{R2})\dot{\delta}_{CR} \\ & - (D_{F1} + D_{F2} + D_S)\dot{y}_F \\ & - (k_{F1} + k_{F2} + k_S)y_F \\ & - (D_{R1} + D_{R2})\dot{y}_R \\ & - (k_{R1} + k_{R2})y_R \\ & - D_S\dot{y}_{DR} \\ & - k_S y_{DR} \\ & - l_{S2}D_S\dot{\delta}_S \\ & - l_{S2}k_S\delta_S = 0 \end{aligned} \quad (7-1-2)$$

Consolidating Eq.(7-1-2);

$$\begin{aligned} & Mv_x\dot{\phi} + Mv_x\dot{\beta} \\ & - (l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S)\dot{\delta}_{CF} \\ & - (l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S)\dot{\delta}_{CF} \\ & - (l_{R1}D_{R1} + l_{R2}D_{R2})\dot{\delta}_{CR} \\ & - (l_{R1}k_{R1} + l_{R2}k_{R2})\dot{\delta}_{CR} \\ & - (D_{F1} + D_{F2} + D_S)\dot{y}_F \\ & - (k_{F1} + k_{F2} + k_S)y_F \\ & - (D_{R1} + D_{R2})\dot{y}_R \\ & - (k_{R1} + k_{R2})y_R \\ & - D_S\dot{y}_{DR} \\ & - k_S y_{DR} \\ & - l_{S2}D_S\dot{\delta}_S \\ & - l_{S2}k_S\delta_S = 0 \end{aligned} \quad (7-1-3)$$

6-2. Equation of sprung mass for rotational motion around the z-axis

Submitting Eq.(7-50-1),(7-51-1),(7-52-1),(7-53-1) and (7-54-1) to Eq.(7-2);

$$\begin{aligned} & I\ddot{\phi} \\ & - x_{F1}\left\{ (y_F + l_{F1}\delta_{CF})k_{F1} + (\dot{y}_F + l_{F1}\dot{\delta}_{CF})D_{F1} \right\} \\ & - x_{F2}\left\{ (y_F + l_{F2}\delta_{CF})k_{F2} + (\dot{y}_F + l_{F2}\dot{\delta}_{CF})D_{F2} \right\} \\ & - x_{R1}\left\{ (y_R + l_{R1}\delta_{CR})k_{R1} + (\dot{y}_R + l_{R1}\dot{\delta}_{CR})D_{R1} \right\} \\ & - x_{R2}\left\{ (y_R + l_{R2}\delta_{CR})k_{R2} + (\dot{y}_R + l_{R2}\dot{\delta}_{CR})D_{R2} \right\} \\ & - x_S\left\{ (y_F + l_{FS}\delta_{CF} + l_{S2}\delta_S + y_{DR})k_S + (\dot{y}_F + l_{FS}\dot{\delta}_{CF} + l_{S2}\dot{\delta}_S + \dot{y}_{DR})D_S \right\} = 0 \end{aligned} \quad (7-2-1)$$

Consolidating Eq.(7-2-1) regarding $\varphi, \delta_{CF}, \delta_{CR}, y_F, y_R$ and δ_S ;

$$\begin{aligned} & I\ddot{\phi} \\ & - (x_{F1}l_{F1}D_{F1} + x_{F2}l_{F2}D_{F2} + x_Sl_{FS}D_S)\dot{\delta}_{CF} \\ & - (x_{F1}l_{F1}k_{F1} + x_{F2}l_{F2}k_{F2} + x_Sl_{FS}k_S)\dot{\delta}_{CF} \\ & - (x_{F1}D_{F1} + x_{F2}D_{F2} + x_S D_S)\dot{y}_F \\ & - (x_{F1}k_{F1} + x_{F2}k_{F2} + x_S k_S)y_F \\ & - (x_{R1}l_{R1}D_{R1} + x_{R2}l_{R2}D_{R2})\dot{\delta}_{CR} \\ & - (x_{R1}l_{R1}k_{R1} + x_{R2}l_{R2}k_{R2})\dot{\delta}_{CR} \\ & - (x_{R1}D_{R1} + x_{R2}D_{R2})\dot{y}_{DR} \\ & - (x_{R1}k_{R1} + x_{R2}k_{R2})y_{DR} \\ & - x_S D_S\dot{y}_{DR} \\ & - x_S k_S y_{DR} \\ & - (x_S l_{S2} D_S)\dot{\delta}_S \\ & - (x_S l_{S2} k_S)\delta_S = 0 \end{aligned} \quad (7-2-2)$$

6-3. Equation of front un-sprung mass for lateral motion in the y_i -axis direction

Submitting Eq.(7-15),(7-16),(7-50-1) and (7-51-1) to Eq.(7-3);

$$\begin{aligned}
 & M_F(\ddot{y}_F + \dot{v}_y + x_F\ddot{\varphi} + v_x\dot{\varphi} + v_x\dot{\delta}_{CF}) \\
 & + \{(y_F + l_{F1}\delta_{CF})k_{F1} + (\dot{y}_F + l_{F1}\dot{\delta}_{CF})D_{F1}\}\cos\delta_{CF} \quad (7-3-1) \\
 & + \{(y_F + l_{F2}\delta_{CF})k_{F2} + (\dot{y}_F + l_{F2}\dot{\delta}_{CF})D_{F2}\}\cos\delta_{CF} \\
 & + F_{ys} = 0
 \end{aligned}$$

Consolidating Eq.(7-3-1) regarding $\varphi, \beta, \delta_{CF}, y_F$ and δ_S putting $\cos\delta_{CF} \approx 1, v_y = \beta v_x$;

$$\begin{aligned}
 & M_F x_F \ddot{\varphi} \\
 & + M_F v_x \dot{\varphi} \\
 & + M_F v_x \dot{\beta} \\
 & + (M_F v_x + l_{F1}D_{F1} + l_{F2}D_{F2})\dot{\delta}_{CF} \\
 & + (l_{F1}k_{F1} + l_{F2}k_{F2})\delta_{CF} \quad (7-3-2) \\
 & + M_F \ddot{y}_F \\
 & + (D_{F1} + D_{F2})\dot{y}_F \\
 & + (k_{F1} + k_{F2})y_F \\
 & + F_{ys} = 0
 \end{aligned}$$

Substituting F_{ys} of Eq.(7-7) to F_{ys} of (7-3-2);

$$\begin{aligned}
 & M_F x_F \ddot{\varphi} \\
 & + M_F v_x \dot{\varphi} \\
 & + M_F v_x \dot{\beta} \\
 & + (M_F v_x + l_{F1}D_{F1} + l_{F2}D_{F2})\dot{\delta}_{CF} \\
 & + (l_{F1}k_{F1} + l_{F2}k_{F2})\delta_{CF} \\
 & + M_F \ddot{y}_F \\
 & + (D_{F1} + D_{F2})\dot{y}_F \\
 & + (k_{F1} + k_{F2})y_F \\
 & + \{M_S(\dot{v}_{y3} + v_{x3}\dot{\varphi}_3) - F_{yST} \cos(\delta_S + \delta_{CF}) + SF_F\} / \cos\delta_S = 0 \quad (7-3-3)
 \end{aligned}$$

Substituting SF_F of Eq.(7-30) to Eq.(7-3-3);

$$\begin{aligned}
 & M_F x_F \ddot{\varphi} \\
 & + M_F v_x \dot{\varphi} \\
 & + M_F v_x \dot{\beta} \\
 & + (M_F v_x + l_{F1}D_{F1} + l_{F2}D_{F2})\dot{\delta}_{CF} \\
 & + (l_{F1}k_{F1} + l_{F2}k_{F2})\delta_{CF} \\
 & + M_F \ddot{y}_F \\
 & + (D_{F1} + D_{F2})\dot{y}_F \\
 & + (k_{F1} + k_{F2})y_F \\
 & + [M_S(\dot{v}_{y3} + v_{x3}\dot{\varphi}_3) - F_{yST} \cos(\delta_S + \delta_{CF}) \\
 & + FC_P \{\dot{y}_F + v_y + (x_F + l_{FT})\dot{\varphi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S\} / v_x] / \cos\delta_S \\
 & = 0 \quad (7-3-4)
 \end{aligned}$$

Substituting Eq.(7-27) to v_{y3} , Eq.(7-18) to φ_3 and Eq.(7-51-1) to F_{yST} of Eq.(7-3-4);

$$\begin{aligned}
 & M_F x_F \ddot{\varphi} \\
 & + M_F v_x \dot{\varphi} \\
 & + M_F v_x \dot{\beta} \\
 & + (M_F v_x + l_{F1}D_{F1} + l_{F2}D_{F2})\dot{\delta}_{CF} \\
 & + (l_{F1}k_{F1} + l_{F2}k_{F2})\delta_{CF} \\
 & + M_F \ddot{y}_F \\
 & + (D_{F1} + D_{F2})\dot{y}_F \\
 & + (k_{F1} + k_{F2})y_F \\
 & + [M_S(\dot{y}_F + \dot{v}_y + x_F\ddot{\varphi} + l_{FS}\ddot{\delta}_{CF} + l_{S2}\ddot{\delta}_S + v_{x3}\dot{\varphi} + v_{x3}\dot{\delta}_{CF} + v_{x3}\dot{\delta}_S) \\
 & + \{(y_F + l_{FS}\delta_{CF} + l_{S2}\delta_S + y_{DR})k_S + (\dot{y}_F + l_{FS}\dot{\delta}_{CF} + l_{S2}\dot{\delta}_S + \dot{y}_{DR})D_S\}\cos(\delta_S + \delta_{CF}) \\
 & + FC_P \{\dot{y}_F + v_y + (x_F + l_{FT})\dot{\varphi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S\} / v_x] / \cos\delta_S \\
 & = 0 \quad (7-3-5)
 \end{aligned}$$

Consolidating Eq.(7-3-5) putting $v_{x3} \approx v_x, \cos\delta_S \approx 1, \cos(\delta_F + \delta_{CF}) \approx 1$, then putting $v_y = v_x \beta$, that is, $\dot{v}_y = \dot{v}_x \beta + v_x \dot{\beta} \approx v_x \dot{\beta}$;

$$\begin{aligned}
 & (M_F + M_S)x_F \ddot{\varphi} \\
 & + \{(M_F + M_S)v_x + (x_F + l_{FT})FC_P / v_x\}\dot{\varphi} \\
 & + (M_F + M_S)v_x \dot{\beta} \\
 & + FC_P \beta \\
 & + M_S l_{FS} \dot{\delta}_{CF} \\
 & + \{(M_F + M_S)v_x + l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S + FC_P(l_{FS} + l_{FT}) / v_x\}\dot{\delta}_{CF} \\
 & + (l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S)\delta_{CF} \\
 & + (M_F + M_S)\ddot{y}_F \\
 & + (D_{F1} + D_{F2} + D_S + FC_P / v_x)\dot{y}_F \\
 & + (k_{F1} + k_{F2} + k_S)y_F \\
 & + M_S l_{S2} \dot{\delta}_S \\
 & + \{M_S v_x + l_{S2}D_S + FC_P(l_{S2} + l_{FT}) / v_x\}\dot{\delta}_S \\
 & + l_{S2}k_S \delta_S \\
 & = -D_S \dot{y}_{DR} - k_S y_{DR} \quad (7-3-6)
 \end{aligned}$$

6-4. Equation of front un-sprung mass for rotational motion around the z_i-axis

Substituting Eq.(7-16),(7-50-1),(7-51-1) and (7-32) to Eq.(7-4);

$$\begin{aligned}
 & I_F (\ddot{\phi} + \ddot{\delta}_{CF}) \\
 & + l_{F1} \{ (y_F + l_{F1} \delta_{CF}) k_{F1} + (\dot{y}_F + l_{F1} \dot{\delta}_{CF}) D_{F1} \} \cos \delta_{CF} \\
 & + l_{F2} \{ (y_F + l_{F2} \delta_{CF}) k_{F2} + (\dot{y}_F + l_{F2} \dot{\delta}_{CF}) D_{F2} \} \cos \delta_{CF} \\
 & - (k_{FS} \delta_S + D_{FS} \dot{\delta}_S) \\
 & + l_{FS} F_{yS} = 0
 \end{aligned} \tag{7-4-1}$$

Consolidating Eq.(7-4-1) putting $\cos \delta_{CF} \approx 1$, $\cos \delta_{CR} \approx 1$;

$$\begin{aligned}
 & I_F \ddot{\phi} \\
 & + I_F \ddot{\delta}_{CF} \\
 & + (l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2}) \dot{\delta}_{CF} \\
 & + (l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2}) \delta_{CF} \\
 & + (l_{F1} D_{F1} + l_{F2} D_{F2}) \dot{y}_F \\
 & + (l_{F1} k_{F1} + l_{F2} k_{F2}) y_F \\
 & - D_{FS} \dot{\delta}_S \\
 & - k_{FS} \delta_S \\
 & + l_{FS} F_{yS} = 0
 \end{aligned} \tag{7-4-2}$$

Substituting F_{yS} of Eq.(7-7) to F_{yS} of Eq.(7-4-2);

$$\begin{aligned}
 & I_F \ddot{\phi} \\
 & + I_F \ddot{\delta}_{CF} \\
 & + (l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2}) \dot{\delta}_{CF} \\
 & + (l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2}) \delta_{CF} \\
 & + (l_{F1} D_{F1} + l_{F2} D_{F2}) \dot{y}_F \\
 & + (l_{F1} k_{F1} + l_{F2} k_{F2}) y_F \\
 & - D_{FS} \dot{\delta}_S \\
 & - k_{FS} \delta_S \\
 & + l_{FS} \{ M_S (\dot{v}_{y3} + v_{x3} \dot{\phi}_3) - F_{yST} \cos(\delta_S + \delta_{CF}) + SF_F \} / \cos \delta_S = 0
 \end{aligned} \tag{7-4-3}$$

Substituting Eq.(7-30) to SF_F of Eq.(7-4-3);

$$\begin{aligned}
 & I_F \ddot{\phi} \\
 & + I_F \ddot{\delta}_{CF} \\
 & + (l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2}) \dot{\delta}_{CF} \\
 & + (l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2}) \delta_{CF} \\
 & + (l_{F1} D_{F1} + l_{F2} D_{F2}) \dot{y}_F \\
 & + (l_{F1} k_{F1} + l_{F2} k_{F2}) y_F \\
 & - D_{FS} \dot{\delta}_S \\
 & - k_{FS} \delta_S \\
 & + l_{FS} [M_S (\dot{v}_{y3} + v_{x3} \dot{\phi}_3) - F_{yST} \cos(\delta_S + \delta_{CF}) \\
 & + FC_P \{ \dot{y}_F + v_y + (x_F + l_{FT}) \dot{\phi} + (l_{FS} + l_{FT}) \dot{\delta}_{CF} + (l_{S2} + l_{FT}) \dot{\delta}_S \} / v_x] / \cos \delta_S = 0
 \end{aligned} \tag{7-4-4}$$

Submitting F_{yST} of Eq.(7-52-1) to V_{y3} and $\dot{\phi}_3$ of Eq.(7-52-1);

$$\begin{aligned}
 & I_F \ddot{\phi} \\
 & + I_F \ddot{\delta}_{CF} \\
 & + (l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2}) \dot{\delta}_{CF} \\
 & + (l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2}) \delta_{CF} \\
 & + (l_{F1} D_{F1} + l_{F2} D_{F2}) \dot{y}_F \\
 & + (l_{F1} k_{F1} + l_{F2} k_{F2}) y_F \\
 & - D_{FS} \dot{\delta}_S \\
 & - k_{FS} \delta_S \\
 & + l_{FS} [M_S (\dot{y}_F + \dot{v}_y + x_F \ddot{\phi} + l_{FS} \ddot{\delta}_{CF} + l_{S2} \ddot{\delta}_S + v_{x3} \dot{\phi} + v_{x3} \dot{\delta}_{CF} + v_{x3} \dot{\delta}_S) \\
 & + \{ (y_F + l_{FS} \delta_{CF} + l_{S2} \delta_S + y_{DR}) k_S + (\dot{y}_F + l_{FS} \dot{\delta}_{CF} + l_{S2} \dot{\delta}_S + \dot{y}_{DR}) D_S \} \cos(\delta_S + \delta_{CF}) \\
 & + FC_P \{ \dot{y}_F + v_y + (x_F + l_{FT}) \dot{\phi} + (l_{FS} + l_{FT}) \dot{\delta}_{CF} + (l_{S2} + l_{FT}) \dot{\delta}_S \} / v_x] / \cos \delta_S = 0
 \end{aligned} \tag{7-4-5}$$

Consolidating Eq.(7-4-5) putting $v_{x3} \approx v_x$, $\cos \delta_S$

≈ 1 , $\cos(\delta_S + \delta_{CF}) \approx 1$, $v_y = v_x \beta$, That is, $\dot{v}_y = \dot{v}_x \beta + v_x \dot{\beta} \approx v_x \dot{\beta}$;

$$\begin{aligned}
 & (I_F + l_{FS} x_F M_S) \ddot{\phi} \\
 & + \{ l_{FS} v_x M_S + l_{FS} (x_F + l_{FT}) FC_P / v_x \} \dot{\phi} \\
 & + (I_F + l_{FS}^2 M_S) \ddot{\delta}_{CF} \\
 & + \{ l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} + l_{FS}^2 D_S + l_{FS} x_x M_S + l_{FS} (l_{FS} + l_{FT}) FC_P / v_x \} \dot{\delta}_{CF} \\
 & + (l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} + l_{FS}^2 k_S) \delta_{CF} \\
 & + l_{FS} M_S \dot{y}_F \\
 & + (l_{F1} D_{F1} + l_{F2} D_{F2} + l_{FS} D_S + l_{FS} FC_P / v_x) \dot{y}_F \\
 & + (l_{F1} k_{F1} + l_{F2} k_{F2} + l_{FS} D_S) y_F \\
 & + l_{FS} l_{S2} M_S \ddot{\delta}_S \\
 & + \{ l_{FS} M_S v_x + l_{FS} l_{S2} D_S + (l_{S2} + l_{FT}) FC_P / v_x - D_{FS} \} \dot{\delta}_S \\
 & + (l_{FS} l_{S2} k_S - k_{FS}) \delta_S \\
 & + l_{FS} v_x M_S \dot{\beta} \\
 & + l_{FS} FC_P \beta \\
 & = -l_{FS} D_S \dot{y}_{DR} - l_{FS} k_S y_{DR}
 \end{aligned} \tag{7-4-6}$$

6-5. Equation of rear un-sprung mass for lateral motion in the y_2 -axis direction

Submitting Eq.(7-24),(7-25),(7-53-1),(7-54-1) and (7-31) to Eq.(7-5);

$$\begin{aligned} & M_R(\ddot{y}_R + \dot{v}_y + x_R\ddot{\phi} + v_x\dot{\phi} + v_x\dot{\delta}_{CR}) \\ & + \{(y_D + l_{R1}\delta_{CR})k_{R1} + (\dot{y}_R + l_{R1}\dot{\delta}_{CR})D_{R1}\}\cos\delta_{CR} \quad (7-5-1) \\ & + \{(y_D + l_{R2}\delta_{CR})k_{R2} + (\dot{y}_R + l_{R2}\dot{\delta}_{CR})D_{R2}\}\cos\delta_{CR} \\ & + RC_P\{\dot{y}_R + v_y + (x_R + l_{RT})\dot{\phi} + l_{RT}\dot{\delta}_{CR}\}/v_x = 0 \end{aligned}$$

Consolidating Eq.(7-5-1) putting $\cos\delta_{CR} \approx 1, v_y = v_x\beta$;

$$\begin{aligned} & M_R x_R \ddot{\phi} \\ & + \{M_R v_x + RC_P(x_R + l_{RT})/v_x\}\dot{\phi} \\ & + M_R v_x \dot{\beta} \\ & + RC_P \beta \\ & + (M_R v_x + l_{R1}D_{R1} + l_{R2}D_{R2} + RC_P l_{RT}/v_x)\dot{\delta}_{CR} \quad (7-5-2) \\ & + (l_{R1}k_{R1} + l_{R2}k_{R2})\delta_{CR} \\ & + M_R \ddot{y}_R \\ & + (D_{R1} + D_{R2} + RC_P/v_x)\dot{y}_R \\ & + (k_{R1} + k_{R2})y_R = 0 \end{aligned}$$

6-6. Equation of rear un-sprung mass for rotational motion around the z_2 -axis

Submitting Eq.(7-25),(7-53-1),(7-54-1) and (7-31) to Eq.(7-6);

$$\begin{aligned} & I_R(\ddot{\phi} + \ddot{\delta}_{CR}) \\ & + l_{R1}\{(y_R + l_{R1}\delta_{CR})k_{R1} + (\dot{y}_R + l_{R1}\dot{\delta}_{CR})D_{R1}\} \quad (7-6-1) \\ & + l_{R2}\{(y_R + l_{R2}\delta_{CR})k_{R2} + (\dot{y}_R + l_{R2}\dot{\delta}_{CR})D_{R2}\} \\ & + l_{RT}RC_P\{\dot{y}_R + v_y + (x_R + l_{RT})\dot{\phi} + l_{RT}\dot{\delta}_{CR}\}/v_x = 0 \end{aligned}$$

Consolidating Eq.(7-6-1) putting $v_y = v_x\beta$;

$$\begin{aligned} & I_R \ddot{\phi} \\ & + l_{RT}RC_P(x_R + l_{RT})/v_x \dot{\phi} \\ & + l_{RT}RC_P \beta \\ & + I_R \ddot{\delta}_{CR} \\ & + (l_{R1}^2 D_{R1} + l_{R2}^2 D_{R2} + l_{RT}^2 RC_P/v_x)\dot{\delta}_{CR} \quad (7-6-2) \\ & + (l_{R1}^2 k_{R1} + l_{R2}^2 k_{R2})\delta_{CR} \\ & + (l_{R1}D_{R1} + l_{R2}D_{R2} + l_{RT}RC_P/v_x)\dot{y}_R \\ & + (l_{R1}k_{R1} + l_{R2}k_{R2})y_R = 0 \end{aligned}$$

6-7. Equation of steering system mass for rotational motion around the z_3 -axis

Substituting the reactive steering force F_{yST} of Eq.(7-7) to F_{yST} of Eq.(7-8);

$$\begin{aligned} & I_S \ddot{\phi}_3 - l_{S2}F_{yST} \cos(\delta_S + \delta_{CF}) + l_{FT}SF_F + T_{FS} \\ & - l_{S1}\{M_S(\dot{v}_{y3} + v_{y3}\dot{\phi}_3) - F_{yST} \cos(\delta_S + \delta_{CF}) + SF_F\} = 0 \quad (7-8-1) \end{aligned}$$

Consolidating Eq.(7-8-1);

$$\begin{aligned} & I_S \ddot{\phi}_3 - l_{S1}M_3(v_{y3} + v_{x3}\dot{\phi}_3) + (l_{S1} - l_{S2})\{F_{yST} \cos(\delta_S + \delta_{CF})\} \\ & + (L_{FT} - l_{S1})SF_F + T_{FS} = 0 \quad (7-8-2) \end{aligned}$$

Submitting Eq.(7-30) and Eq.(7-32) to SF_F and T_{FS} of Eq.(7-8-2);

$$\begin{aligned} & I_S \ddot{\phi}_3 - l_{S1}M_3(v_{y3} + v_{x3}\dot{\phi}_3) + (l_{S1} - l_{S2})\{F_{yST} \cos(\delta_S + \delta_{CF})\} \\ & + (L_{FT} - l_{S1})\{\dot{y}_F + v_y + (x_F + l_{FT})\dot{\phi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S\}FC_P/v_x \\ & + (k_{FS}\delta_S + D_{FS}\dot{\delta}_S) = 0 \quad (7-8-3) \end{aligned}$$

Submitting Eq.(7-27) and (7-18) to v_{y3} and F_{yST} of Eq.(7-8-3);

$$\begin{aligned} & I_S(\ddot{\phi} + \ddot{\delta}_{CF} + \ddot{\delta}_S) \\ & - l_{S1}M_3(\dot{y}_F + \dot{v}_y + x_F\ddot{\phi} + l_{FS}\ddot{\delta}_{CF} + l_{S2}\ddot{\delta}_S + v_{x3}\dot{\phi} + v_{x3}\dot{\delta}_{CF} + v_{x3}\dot{\delta}_S) \\ & - (l_{S1} - l_{S2})\{(y_F + l_{FS}\delta_{CF} + l_{S2}\delta_S + y_{DR})k_S + (\dot{y}_F + l_{FS}\dot{\delta}_{CF} + l_{S2}\dot{\delta}_S + \dot{y}_{DR})D_S\}\cos(\delta_S + \delta_{CF}) \\ & + (l_{FT} + l_{S1})\{\dot{y}_F + v_y + (x_F + l_{FT})\dot{\phi} + (l_{FS} + l_{FT})\dot{\delta}_{CF} + (l_{S2} + l_{FT})\dot{\delta}_S\}FC_P/v_x \\ & + (k_{FS}\delta_S + D_{FS}\dot{\delta}_S) = 0 \quad (7-8-4) \end{aligned}$$

Consolidating Eq.(7-8-4) putting $v_y = v_x\beta, v_{x3} \approx v_x, \cos(\delta_S + \delta_{CF}) \approx 1$;

$$\begin{aligned} & (I_S - l_{S1}x_F M_3)\ddot{\phi} \\ & + \{-l_{S1}M_3 v_x + (l_{FT} + l_{S1})(x_F + l_{FT})FC_P/v_x\}\dot{\phi} \\ & - l_{S1}M_3 v_x \dot{\beta} \\ & + (l_{FT} + l_{S1})FC_P \beta \\ & + (I_S - l_{S1}l_{FS}M_3)\ddot{\delta}_{CF} \\ & + \{-l_{S1}M_3 v_x - (l_{S1} - l_{S2})l_{FS}D_S + (l_{FT} + l_{S1})(l_{FS} + l_{FT})FC_P/v_x\}\dot{\delta}_{CF} \\ & - (l_{S1} - l_{S2})l_{FS}k_S \delta_{CF} \\ & - l_{S1}M_3 \ddot{y}_F \\ & + \{- (l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})FC_P/v_x\}\dot{y}_F \\ & - (l_{S1} - l_{S2})k_S y_F \\ & + (I_S - l_{S1}l_{S2}M_3)\ddot{\delta}_S \\ & + \{-l_{S1}M_3 v_x - (l_{S1} - l_{S2})l_{S2}D_S + (l_{FT} + l_{S1})(l_{S2} + l_{FT})FC_P/v_x + D_{FS}\}\dot{\delta}_S \\ & + \{- (l_{S1} - l_{S2})l_{S2}k_S + k_{FS}\}\delta_S \\ & = (l_{S1} - l_{S2})D_S \dot{y}_{DR} + (l_{S1} - l_{S2})k_S y_{DR} \quad (7-8-5) \end{aligned}$$

7. MATRIX OF THE DEPLOYED FUNDAMENTAL MOTION EQUATIONS

The deployed fundamental equations, that is, Eq.(7-2-2) for $\dot{\phi}$, Eq.(7-1-3) for β , Eq.(7-4-6) for δ_{CF} , Eq.(7-3-6) for y_F , Eq.(7-6-2) for δ_{CR} , Eq.(7-5-2) for y_R and Eq.(7-8-5) for δ_S , are described with regard to the matrix as shown in Eq.(7-55);

$$\begin{bmatrix} c_{12}D^2 & 0 & c_{31}D + c_{30} & c_{41}D + c_{40} & c_{51}D + c_{50} & c_{61}D + c_{60} & c_{71}D + c_{70} \\ d_{11}D & d_{21}D & d_{31}D + d_{30} & d_{41}D + d_{40} & 0 & d_{61}D + d_{60} & d_{71}D + c_{70} \\ e_{12}D^2 & 0 & e_{32}D^2 + e_{31}D + e_{30} & e_{41}D + e_{40} & 0 & 0 & e_{71}D + e_{70} \\ f_{12}D^2 + f_{11}D & f_{21}D & f_{31}D + f_{30} & f_{42}D^2 + f_{41}D + f_{40} & 0 & 0 & f_{71}D + f_{70} \\ g_{12}D^2 + g_{11}D & g_{20} & 0 & 0 & g_{52}D^2 + g_{51}D + g_{50} & g_{61}D + g_{60} & 0 \\ h_{12}D^2 + h_{11}D & h_{21}D + h_{20} & 0 & 0 & h_{51}D + h_{50} & h_{62}D^2 + h_{61}D + h_{60} & 0 \\ s_{11}D + s_{10} & s_{21}D + s_{20} & s_{32}D^2 + s_{31}D + s_{30} & s_{42}D^2 + s_{41}D + s_{40} & 0 & 0 & s_{72}D^2 + s_{71}D + s_{70} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \beta \\ \delta_{CF} \\ y_F \\ \delta_{CR} \\ y_R \\ \delta_S \end{bmatrix} = \begin{bmatrix} c_{81}D + c_{80} \\ d_{81}D + d_{80} \\ e_{81}D + e_{80} \\ f_{81}D + f_{80} \\ 0 \\ 0 \\ s_{81}D + s_{80} \end{bmatrix} y_{DR} \quad (7-55)$$

Where, "D" is the differential calculus operator. The contents of c_{11}, \dots, s_{80} are shown as follows;

$$\begin{aligned} c_{11} &= I \\ c_{31} &= -(x_{F1}l_{F1}D_{F1} + x_{F2}l_{F2}D_{F2} + x_S l_{FS} D_S) \\ c_{30} &= -(x_{F1}l_{F1}k_{F1} + x_{F2}l_{F2}k_{F2} + x_S l_{FS} k_S) \\ c_{41} &= -(x_{F1}D_{F1} + x_{F2}D_{F2} + x_S D_S) \\ c_{40} &= -(x_{F1}k_{F1} + x_{F2}k_{F2} + x_S k_S) \\ c_{51} &= -(x_{R1}l_{R1}D_{R1} + x_{R2}l_{R2}D_{R2}) \\ c_{50} &= -(x_{R1}l_{R1}k_{R1} + x_{R2}l_{R2}k_{R2}) \\ c_{61} &= -(x_{R1}D_{R1} + x_{R2}D_{R2}) \\ c_{60} &= -(x_{R1}k_{R1} + x_{R2}k_{R2}) \\ c_{71} &= -x_S l_{S2} D_S \\ c_{70} &= -x_S l_{S2} k_S \\ c_{81} &= +x_S D_S \\ c_{80} &= +x_S k_S \end{aligned} \quad (7-55-1)$$

$$\begin{aligned} d_{10} &= Mv_x \\ d_{21} &= Mv_x \\ d_{31} &= -(l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S) \\ d_{30} &= -(l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S) \\ d_{41} &= -(D_{F1} + D_{F2} + D_S) \\ d_{40} &= -(k_{F1} + k_{F2} + k_S) \\ d_{51} &= -(l_{R1}D_{R1} + l_{R2}D_{R2}) \\ d_{50} &= -(l_{R1}k_{R1} + l_{R2}k_{R2}) \\ d_{61} &= -(D_{R1} + D_{R2}) \\ d_{60} &= -(k_{R1} + k_{R2}) \\ d_{71} &= -l_{S2}D_S \\ d_{70} &= -l_{S2}k_S \\ d_{81} &= +D_S \\ d_{80} &= +k_S \end{aligned} \quad (7-55-2)$$

$$\begin{aligned}
e_{11} &= +I_F + l_{FS}x_F M_S \\
e_{10} &= +l_{FS}v_x M_S + l_{FS}(x_F + l_{FT})FC_P / v_x \\
e_{21} &= +l_{FS}v_x M_S \\
e_{20} &= +l_{FS}FC_P \\
e_{32} &= +I_F + l_{FS}^2 M_S \\
e_{31} &= +l_{F1}^2 D_{F1} + l_{F2}^2 D_{F2} + l_{FS}^2 D_S + l_{FS}v_x M_S + l_{FS}(l_{FS} + l_{FT})FC_P / v_x \\
e_{30} &= +l_{F1}^2 k_{F1} + l_{F2}^2 k_{F2} + l_{FS}^2 k_S \\
e_{42} &= +l_{FS}M_S \\
e_{41} &= +l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S + l_{FS}FC_P / v_x \\
e_{40} &= +l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S \\
e_{72} &= +l_{FS}l_{S2}M_S \\
e_{71} &= +l_{FS}v_x M_S + l_{FS}l_{S2}D_S + (l_{S2} + l_{FT})FC_P / v_x - D_{FS} \\
e_{70} &= +l_{FS}l_{S2}D_S - k_{FS} \\
e_{81} &= -l_{FS}D_S \\
e_{80} &= -l_{FS}k_S
\end{aligned} \tag{7-55-3}$$

$$\begin{aligned}
f_{11} &= +(M_F + M_S)x_F \\
f_{10} &= +(M_F + M_S)v_x + (x_F + l_{FT})FC_P / v_x \\
f_{21} &= +(M_F + M_S)v_x \\
f_{20} &= +FC_P \\
f_{32} &= +l_{FS}M_S \\
f_{31} &= +(M_F + M_S)v_x + l_{F1}D_{F1} + l_{F2}D_{F2} + l_{FS}D_S + (l_{FS} + l_{FT})FC_P / v_x \\
f_{30} &= +l_{F1}k_{F1} + l_{F2}k_{F2} + l_{FS}k_S \\
f_{42} &= +M_F + M_S \\
f_{41} &= +D_{F1} + D_{F2} + D_S + FC_P / v \\
f_{40} &= +k_{F1} + k_{F2} + k_S \\
f_{72} &= +l_{S2}M_S \\
f_{71} &= +M_S v_x + l_{S2}D_S + (l_{S2} + l_{FT})FC_P / v_x \\
f_{70} &= +l_{S2}k_S \\
f_{81} &= +D_S \\
f_{80} &= +k_S
\end{aligned} \tag{7-55-4}$$

$$\begin{aligned}
g_{11} &= I_R \\
g_{10} &= +l_{RT}(x_R + l_{RT})RC_P / v_x \\
g_{20} &= +l_{RT}RC_P \\
g_{52} &= I_R \\
g_{51} &= +l_{R1}^2 D_{R1} + l_{R2}^2 D_{R2} \\
g_{50} &= +l_{R1}^2 k_{R1} + l_{R2}^2 k_{R2} \\
g_{61} &= +l_{R1}D_{R1} + l_{R2}D_{R2} + l_{RT}RC_P / v_x \\
g_{60} &= +l_{R1}k_{R1} + l_{R2}k_{R2}
\end{aligned} \tag{7-55-5}$$

$$\begin{aligned}
h_{11} &= x_R M_x \\
h_{10} &= M_R v_x + (x_R + l_{RT})RC_P / v_x \\
h_{21} &= M_R v_x \\
h_{20} &= +RC_P \\
h_{51} &= M_R v_x + l_{R1}D_{R1} + l_{R2}D_{R2} + l_{RT}RC_P / v_x \\
h_{50} &= +l_{R1}k_{R1} + l_{R2}k_{R2} \\
h_{62} &= M_R \\
h_{61} &= +D_{R1} + D_{R2} + RC_P / v_x \\
h_{60} &= +k_{R1} + k_{R2} \\
s_{11} &= +I_S - l_{S1}x_F M_3 \\
s_{10} &= -l_{S1}M_3 v_x + (l_{FT} + l_{S1})(x_F + l_{FT})FC_P / v_x \\
s_{21} &= -l_{S1}M_3 v_x \\
s_{20} &= +(l_{FT} + l_{S1})FC_P \\
s_{32} &= +I_S - l_{S1}l_{FS}M_3 \\
s_{31} &= -l_{S1}M_3 v_x - l_{FS}(l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})(l_{FS} + l_{FT})FC_P / v_x \\
s_{30} &= -l_{FS}(l_{S1} - l_{S2})k_S \\
s_{42} &= -l_{S1}M_3 \\
s_{41} &= -(l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})FC_P / v_x \\
s_{40} &= -(l_{S1} - l_{S2})k_S \\
s_{72} &= -l_{S1}l_{S2}M_3 \\
s_{71} &= -l_{S1}M_3 v_x - l_{S2}(l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})(l_{S2} + l_{FT})FC_P / v_x + D_{FS} \\
s_{70} &= -l_{FS}(l_{S1} - l_{S2})k_S + k_{FS} \\
s_{81} &= +(l_{S1} - l_{S2})D_S \\
s_{80} &= +(l_{S1} - l_{S2})k_S
\end{aligned} \tag{7-55-6}$$

$$\begin{aligned}
s_{11} &= +I_S - l_{S1}x_F M_3 \\
s_{10} &= -l_{S1}M_3 v_x + (l_{FT} + l_{S1})(x_F + l_{FT})FC_P / v_x \\
s_{21} &= -l_{S1}M_3 v_x \\
s_{20} &= +(l_{FT} + l_{S1})FC_P \\
s_{32} &= +I_S - l_{S1}l_{FS}M_3 \\
s_{31} &= -l_{S1}M_3 v_x - l_{FS}(l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})(l_{FS} + l_{FT})FC_P / v_x \\
s_{30} &= -l_{FS}(l_{S1} - l_{S2})k_S \\
s_{42} &= -l_{S1}M_3 \\
s_{41} &= -(l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})FC_P / v_x \\
s_{40} &= -(l_{S1} - l_{S2})k_S \\
s_{72} &= -l_{S1}l_{S2}M_3 \\
s_{71} &= -l_{S1}M_3 v_x - l_{S2}(l_{S1} - l_{S2})D_S + (l_{FT} + l_{S1})(l_{S2} + l_{FT})FC_P / v_x + D_{FS} \\
s_{70} &= -l_{FS}(l_{S1} - l_{S2})k_S + k_{FS} \\
s_{81} &= +(l_{S1} - l_{S2})D_S \\
s_{80} &= +(l_{S1} - l_{S2})k_S
\end{aligned} \tag{7-55-7}$$

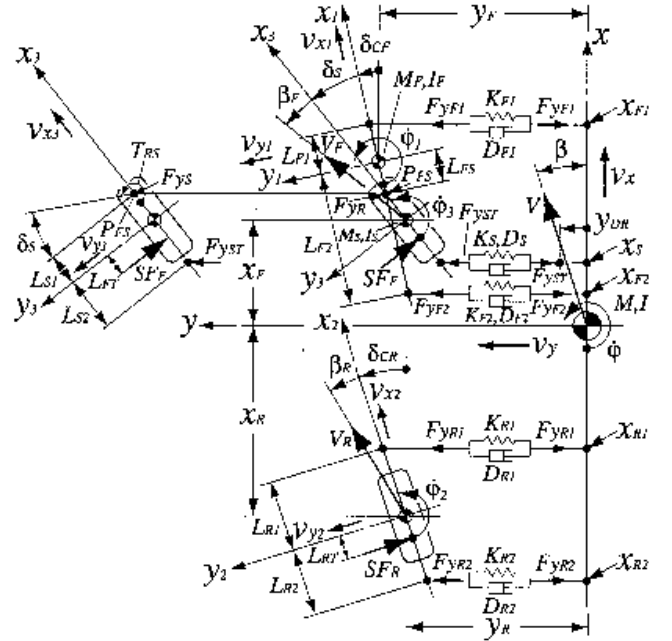


Fig.3-1 Details of 7DOF Model shown as Fig.3

SYMBOLS;

<CONSTANTS>

M : Sprung mass (kg)

M_F : Un-sprung mass of front suspension (kg)

M_R : Un-sprung mass of rear suspension (kg)

M_S : Un-sprung mass of steering system around the king pin (kg)

J : Yawing moment of inertia of sprung mass (kg-m²)

J_F : Yawing moment of inertia of un-sprung mass of front suspension (kg-m²)

J_R : Yawing moment of inertia of un-sprung mass of rear suspension (kg-m²)

J_S : Moment of inertia of steering system around the king pin (kg-m²)

x_F : x value of the location of the front suspension gravity center (m)

x_{F1} : x value of the location of front connecting point of front suspension (m)

x_{F2} : x value of the location of rear connecting point of front suspension (m)

x_S : x value of the location of connecting point of steering system (m)

x_R : x value of the location of the rear suspension gravity center (m)

x_{R1} : x value of the location of front connecting point of rear suspension (m)

x_{R2} : x value of the location of rear connecting point of rear suspension (m)

l_{F1} : x_1 value of the location of front connecting point from front suspension gravity center (m)

l_{F2} : x_1 value of the location of rear connecting point from front suspension gravity center (m)

l_{FS} : x_1 value of the location of king pin (m)

l_{S1} : x_3 value of the location of king pin from steering system gravity center (m)

l_{S2} : x_3 value of the location of connecting point of steering linkage (m)

l_{FT} : x_3 value of the location of applying point of front side force (m)

l_{R1} : x_2 value of the location of front connecting point from rear suspension gravity center (m)

l_{R2} : x_2 value of the location of rear connecting point from rear suspension gravity center (m)

l_{RT} : x_2 value of the location of applying point of rear side force (m)

FC_P : Front cornering power(total of left and right)(N/rad)

RC_P : Rear cornering power(total of left and right)(N/rad)

k_{F1} : Lateral stiffness of front connecting point of front suspension(N/m)

k_{F2} : Lateral stiffness of rear connecting point of front suspension(N/m)

k_{R1} : Lateral stiffness of front connecting point of rear suspension(N/m)

k_{R2} : Lateral stiffness of rear connecting point of rear suspension(N/m)

k_S : Lateral stiffness of steering linkage(N/m)

D_{F1} : Damping coefficient of front connecting point of front suspension(N/m/s)

D_{F2} : Damping coefficient of rear connecting point of front suspension(N/m/s)

D_{R1} : Damping coefficient of front connecting point of rear suspension(N/m/s)

D_{R2} : Damping coefficient of rear connecting point of rear suspension(N/m/s)

D_S : Damping coefficient of steering system(N/m/s)

<VARIABLES>

v_x : x direction velocity of sprung mass(m/s)

v_y : y direction velocity of sprung mass(m/s)

v_{x1} : x_1 direction velocity of un-sprung mass of front suspension(m/s)

v_{y1} : y_1 direction velocity of un-sprung mass of front suspension(m/s)

v_{x2} : x_2 direction velocity of un-sprung mass of rear suspension(m/s)

v_{y2} : y_2 direction velocity of un-sprung mass of rear suspension(m/s)

v_{x3} : x_3 direction velocity of un-sprung mass of steering(m/s)

v_{y3} : y_3 direction velocity of un-sprung mass of steering(m/s)

β : Side slip angle of gravity center of sprung mass(rad)

φ : Yawing angle at gravity center of sprung mass(rad)

β_1 : Side slip angle of gravity center of un-sprung mass of front suspension(rad)

φ_1 : Yawing angle at gravity center of un-sprung mass of front suspension(rad)

β_2 : Side slip angle of gravity center of un-sprung mass of rear suspension(rad)

φ_2 : Yawing angle at gravity center of un-sprung mass of rear suspension(rad)

β_3 : Side slip angle of gravity center of un-sprung mass of steering system(rad)

φ_3 : Yawing angle at gravity center of un-sprung mass of steering system(rad)

δ_{CF} : Compliance steer angle of un-sprung mass of front suspension(rad)

δ_{CR} : Compliance steer angle of un-sprung mass of rear suspension(rad)

δ_s : Compliance steer angle of un-sprung mass of steering system to the front suspension(rad)

y_F : Lateral compliance of front suspension(m)

y_R : Lateral compliance of rear suspension(m)

F_{yF1} : Lateral force between P_{F1} of front suspension and Q_{F1} of sprung mass(N)

F_{yF2} : Lateral force between P_{F2} of front suspension and Q_{F2} of sprung mass(N)

F_{yR1} : Lateral force between P_{R1} of rear suspension and Q_{R1} of sprung mass(N)

F_{yR2} : Lateral force between P_{R2} of rear suspension and Q_{R2} of sprung mass(N)

F_{yST} : Restoring force between P_{ST} of steering system and Q_{ST} of sprung mass(N)

T_{RS} : Restoring torque around the king pin center, P_{FS} (N)

SF_F : Front side force (Total of left and right)(N)

SF_R : Rear side force (Total of left and right)(N)

DF_F : Front drag force (Total of left and right)(N)

DF_R : Rear drag force (Total of left and right)(N)

CF_F : Front cornering force (Total of left and right)(N)

CF_R : Rear cornering force (Total of left and right)(N)

TABLE 1 DATA USED IN CALCULATION OF FIG.4, FIG.5 AND FIG.6

Symbol	Fig.4&6	Fig.5	Symbol	Fig.4	Fig.6	Fig.5
M (kg)	1.3331E04	←	FC_P (N/rad)	3.01E05	←	←
M_F (kg)	3.74E02	←	RC_P (N/rad)	5.41E05	←	←
M_R (kg)	1.400E03	←	k_{F1} (N/m)	1.472E05	←	1.472E05*4
M_S (kg)	5.00E02	←	k_{F2} (N/m)	1.472E05	←	←
J (kg-m ²)	2.11E05	←	k_{FS} (N-m/rad)	0.0	←	←
J_F (kg-m ²)	2.442E02	←	k_{R1} (N/m)	5.992E05	←	2.089E05
J_R (kg-m ²)	1.094E03	←	k_{R2} (N/m)	2.599E06	←	9.349E05
J_S (kg-m ²)	1.77E02	←	k_{ST} (N/m)	1.4295E05	←	←
x_F (m)	4.13	←	D_{F1} (N/m/s)	0.0	6.0E04	←
x_{F1} (m)	4.13+0.35	←	D_{F2} (N/m/s)	0.0	6.0E04	5.0E03
x_{F2} (m)	4.13-0.35	←	D_{FS} (N-m/rad/s)	1.0E04/5	←	←
x_S (m)	4.13-0.165	←	D_{R1} (N/m/s)	0.0	←	4.0E04
x_R (m)	-2.35	←	D_{R2} (N/m/s)	0.0	←	4.0E04
x_{R1} (m)	-2.35+0.716	←	D_{ST} (N/m/s)	0.0	←	←
x_{R2} (m)	-2.35+0.165	-2.35-0.160				
l_{F1} (m)	0.35	←				
l_{F2} (m)	-0.35	←				
l_{FS} (m)	0.0	←				
l_{S1} (m)	-0.15	←				
l_{S2} (m)	-0.15-0.165	←				
l_{FT} (m)	-0.15-0.04	←				
l_{R1} (m)	0.716	←				
l_{R2} (m)	0.165	-0.160				
l_{RT} (m)	0.04	←				