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ABSTRACT

Considerable effort has gone into modelling the performance of the racing car by engineers in professional motorsport teams. The teams are using progressively more sophisticated quasi-static simulations to model vehicle performance. This allows optimisation of vehicle performance to be achieved in a more cost and time effective manner with a more efficient use of physical testing.

Racing cars are driven at the limit of adhesion in the non-linear area of the vehicle's handling performance. Previous simulations have modelled the transient behaviour by approximating it with a quasi-static model which ignores dynamic effects, for example yaw damping. This paper describes a comparison between different cornering modelling strategies, including steady state, quasi-static and transient. The simulation results from the three strategies are compared and evaluated for their ability to model actual racing car behaviour.

INTRODUCTION

The use of a Lap Time Simulation (LTS) package is beneficial and complements the numerous tools (CAD, FEA, CFD) available to the teams and designers. LTS packages allow the team to gain a significant advantage over their competitors. This is achieved by simulating the vehicle negotiating the circuit (which they may have never visited) in any of its possible setup combinations to optimise the vehicle's performance, before they reach the circuit or even before they build the vehicle. LTS packages allows an expensive vehicle to be modelled at its limit of adhesion, without risk of the vehicle being damaged or injury to the driver, as is the case with track testing. Moreover, it has a use during the initial design phase after which parameters, such as the centre of gravity position, cannot be changed. The vehicle can then be produced so that the fundamental design parameters are close to the optimum values.

To simulate a full lap the path of the vehicle is normally split up into segments (for example every 1m) and an analysis made of the vehicle at each segment point, using the external forces acting on the vehicle. This is normally done as a simple quasi-static model, where the circuit is idealised as a series of straights and constant radius turns. A commonly used method of finding the fastest lap time has been described by Milliken et al. [1]. It involves using the corners as limiting factors for the simulation. The maximum speed at which the vehicle can negotiate all the corners is found (which is independent of the straight speeds), this gives the speed the vehicle enters and leaves all the straights. From this, the vehicle's performance along the straights can then be found.

There are several methods by which the vehicle's performance in the corners can be modelled. This paper describes the construction and use of three vehicle modelling strategies which are used to simulate a small formula type racing car in two manoeuvres:

1. The first is a j-turn manoeuvre at constant forward velocity.
2. The second is the vehicle in the same j-turn manoeuvre but where it is braking down from a high forward velocity to allow it to negotiate the corner (i.e. vehicle braking into a hairpin).

The three different vehicle modelling strategies used to find the performance of the vehicle are as follows:

1. Steady state strategy.
2. Quasi-static strategy.
3. Transient strategy.

DEFINITIONS

m: Mass, kg
a: Centre of gravity distance from front axle, m
b: Centre of gravity distance from rear axle, m
C_d: Aerodynamic coefficient of drag
C_l: Aerodynamic coefficient of lift
FA: Frontal area of vehicle, m^2
μ_{rr}: Rolling resistance coefficient
ρ: Air mass density, kg/m^3
I: Second moment of area of vehicle, kgm^2
A: Acceleration, m/s^2
u: Forward velocity, m/s
v: Lateral velocity, m/s
r: Yaw velocity , rad/s
F: Force, N
M: Moment, Nm
N: Normal force on axle, N
D: Drag force, N
α: Tyre slip angle, rad
δ: Steered angle of tyre, rad
τ: Iteration time constant, seconds
x: variable of function f{x}
g: Gravitational constant, m/s^2
l: Wheelbase of car, m
sr: Longitudinal slip ratio at the tyre contact patch

SUBSCRIPTS - ***f:*** At front axle
r: At rear axle
n: Number of iterations
x: In longitudinal direction
y: In lateral direction

Axis System - the axis system used is the SAE standard vehicle and tyre axis system [2].

MODELLING RACING CAR BEHAVIOUR

The equations that are used to model the behaviour of the vehicle can come in many forms but most methods are based on Newton's Laws. These allow prediction of the acceleration of the vehicle due to the forces applied to it. This is an efficient and effective method of assessing the vehicle performance, as simple equations can represent the vehicle in three axis and three rotational degrees of freedom.

To simulate the two different j-turn manoeuvres the vehicle's path is split into two segments. The straight segment where the vehicle is travelling in a straight line and the cornering segment. It is modelled decelerating, from the maximum velocity reached on the straight, using a basic quasi-static braking model, until it is travelling slow enough to allow it to negotiate the corner.

The simulation of the longitudinal acceleration behaviour uses a common method detailed by Gillespie [3]. He suggests that the vehicle's braking behaviour can be idealised using a lumped mass model of the vehicle. The acceleration of the vehicle can then be used to find the longitudinal load transfer, using a moment calculation. Also the effect of aerodynamic lift can be calculated.

To accurately predict behaviour of a racing car, you need to be able to replicate the external forces acting on the vehicle. At low speeds the main external forces acting on the vehicle are generated by the tyres. Racing cars operate at the peak of the tyre force curves, where the force is greatest. The tyres therefore need to be modelled as closely as possible, to allow accurate prediction of racing car The model must not only accurately model tyre performance, but also the effect on resultant force, of changing the normal force at the tyre contact patch.

The most commonly used tyre model for vehicle simulation is based on an empirical approach, often referred to as the Pacejka Magic Formula method [4]. The braking force the tyre produces is a function of the longitudinal slip ratio at the tyre contact patch (which is assumed to be held at the optimum value by the driver) and the normal force on that axle. Other important effects include aerodynamic and tyre drag losses, see equation (1).

$$F_x = f\{sr, N_f, N_r\} + \sum_4 D_{tyre} + D_{aero} = mA_x \quad (1)$$

where $N_f = \frac{mgb}{l} + \frac{mA_x h}{l} + \frac{1}{2}\rho.FA.C_{lf}.u^2$ (2)

and $N_r = \frac{mga}{l} - \frac{mA_x h}{l} + \frac{1}{2}\rho.FA.C_{lr}.u^2$ (3)

The effects of aerodynamic and tyre drag have been found by Dixon [5]. Where for each tyre the drag force is given in equation (4).

$$D_{tyre} = F_y \sin\alpha + \mu_{rr} N \cos\alpha \quad (4)$$

And the total aerodynamic drag force is given in equation (5).

$$D_{aero} = \frac{1}{2}\rho.FA.C_d.u^2 \quad (5)$$

In the cornering segment, the lateral acceleration performance of the vehicle is modelled by assuming the mass of the vehicle is concentrated at the vehicle's centre of gravity, and a suitable value for the second moment of area is given to give a realistic yaw inertia and thus yaw response. Crolla [6] has shown that the equations of motion of the vehicle cornering can be derived from first principles, using a simple inertial axis system, as shown in figure 1.

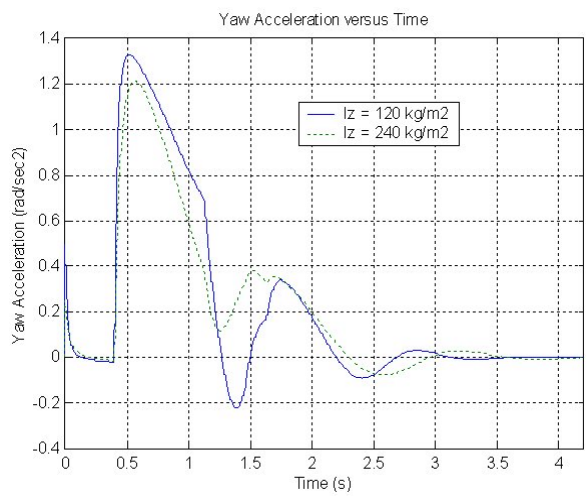

Fig. 1 - The moving axis system, A, for a simple bicycle model.

Applying Newton's second law in the vehicle's lateral and longitudinal directions and about its yaw axis produces equations (6), (7) and (8).

$$m(\dot{u} - vr) = F_{xf} + F_{xr} = \sum F_x \qquad (6)$$

$$m(\dot{v} + ur) = F_{yf} + F_{yr} = \sum F_y \qquad (7)$$

$$I_z \dot{r} = aF_{yf} - bF_{yr} = \sum M_z \qquad (8)$$

The Pacejka Magic Tyre Formula has also been used to derive the lateral force created by the tyre, and is a function of the slip angle at the tyre contact patch and normal force on the tyre. The lateral model has also been extended to take into account the effects of aerodynamics and lateral load transfer to calculate the correct tyre normal force.

The maximum forward velocity that the vehicle can negotiate the corner minimum path radius is found by using a Newton-Rhaphson iteration technique. This approximates the root of the equation as the point where the tangent to the curve at x_n crosses the x-axis. A closer approximation is then made by repeating the process at this new value of x. This is repeated until a close enough approximation is found.

The iteration technique is shown in equation (9), where x_{n+1} is the new solution value based on the previous value x_n.

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \qquad (9)$$

This steer angle is applied in the form of a step input to the lateral acceleration model and waits until the yaw acceleration is zero (the vehicle is undergoing steady state acceleration). The Newton-Rhaphson routine is used to determine what steer angle gives the maximum lateral acceleration at that forward velocity. The forward velocity is increased until no solution can be found or the lateral acceleration limit drops.

For the cornering segment, three different modelling strategies have been used for the comparison study with each using different solution techniques. All three strategies use the same basic models to describe the longitudinal and lateral acceleration performance of the vehicle (detailed above).

STEADY STATE MODELLING STRATEGY - The first model is a simple steady state model where the vehicle's longitudinal and lateral acceleration performance is modelled separately (i.e. the vehicle brakes and then turns in). Thus, only the lateral acceleration performance of the vehicle is taken into account during cornering.

The steady state solution of a simulation occurs when the system is in equilibrium and time dependent variables are zero [6]. This can be found by calculation, for example, by removing all the time dependent variables. It can also be found by simulation with a dynamic model, by leaving the simulation to settle down to its steady state values for a certain forward velocity, after a steer input has been applied to the vehicle.

The cornering values for the simulation are found using the Newton-Rhaphson technique described above. It is modelled negotiating the corner at a fixed velocity, steer angle, path radius and maximum lateral acceleration.

QUASI-STATIC MODELLING STRATEGY – This is similar to the above strategy but this time the corner is split into a series of constant radius turns [7] with decreasing path radius (simulating an increase of steer angle towards the corner apex). There are approximately 50 segments for the corner, making the time step in the approach small, keeping the simulation accurate.

At each path segment (represented by a certain path radius) the vehicle's acceleration is again found by allowing the simulation to settle down to its steady state values. The lateral tyre force needed to maintain this lateral acceleration can be found and using a friction circle approach [1] and the remaining tyre force available is found using a combined Pacejka Magic Formula Tyre model [8]. The remaining tyre force is then used to find the longitudinal acceleration of the vehicle. This ensures that the total combined lateral and longitudinal force generated by the tyre is equivalent to what a real tyre is capable of producing.

The minimum path radius and speed at the apex is found using the Newton-Rhaphson iteration technique (equation (9)) described above and the same models are used to simulate the vehicle's longitudinal (equations (1) to (5)) and lateral (equations (6) to (8)) acceleration behaviour over each path segment.

At present LTS packages find the cornering ability of the vehicle by using the quasi-static solution [9]. This allows the simulation to run quickly and efficiently, as a complicated time dependent solution is not sought.

TRANSIENT MODELLING STRATEGY - In this strategy a transient solution is sought. This is found when the vehicle is undergoing non-steady linear or rotational accelerations [5]. In reality, as the vehicle corners, it is never in a steady state situation as the vehicle is always accelerating in a combination of the linear lateral, longitudinal or normal directions and/or the rotational pitch, roll or yaw directions. The dynamic yaw response of the vehicle and its effects on the overall results is examined in the transient modelling strategy.

The transient simulation takes into account the response time of the vehicle in changing its attitude and direction of travel. Consequently, complex transient simulations can be performed which include the effect of dynamic yaw on the model. Also the lateral and longitudinal weight transfer effects (pitch and roll), as well as slip angle induced tyre drag forces, are passed between the longitudinal (equations (1) to (5)) and lateral (equations (6) to (8)) models, detailed above. Again a Pacejka Magic Force tyre model [8] is used to ensure that the correct combined tyre forces generated are applied.

As the vehicle corners it turns into the corner until it reaches its maximum steer angle and lateral acceleration at the corner apex. The maximum values at the apex which is applied to this model is found using the Newton-Rhaphson iteration technique (equation (9)) described above.

In the second simulated j-turn manoeuvre the vehicle is braking and cornering at the same time. This is modelled for simplicity as if the driver is applying a sinusoidal steer angle and at the same time diminishes the braking force (moving around the edge of the friction circle) until it is zero close to the apex of the corner. Both the quasi-static and transient manoeuvres use the same input values for brake pedal force and steer angle, see figure 2.

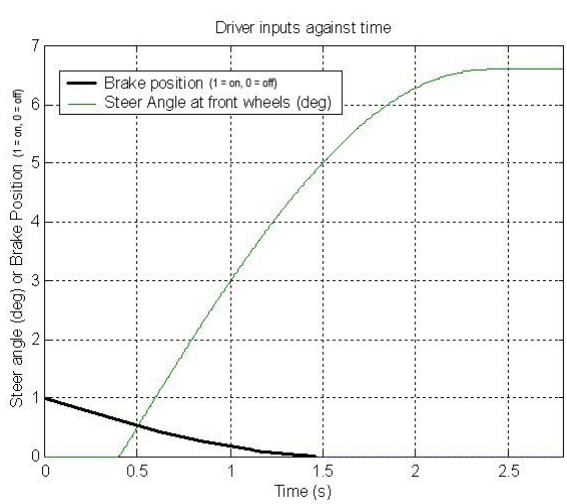

Figure 2 – Driver input values for quasi-static and transient strategies in combined braking and cornering manoeuvre.

RESULTS

The study involves simulating the performance of a small formula car negotiating a tight, 180 degree hairpin up to the corner apex. The vehicle is a formula SAE/student car which is raced at on tight twisty circuits and as such does not have any significant aerodynamic devices, due to the low speeds involved. The vehicle also has enough power to overcome the aerodynamic and tyre drag forces as it negotiates the corner, making the vehicle's performance limited by the frictional force available from the tyres.

The vehicle is simulated in two different manoeuvres, using three different modelling strategies detailed above for both manoeuvres. The first manoeuvre is the vehicle at constant forward velocity, the second is the vehicle braking down from 30 m/s until it is travelling slow enough to allow it to negotiate the corner at maximum lateral acceleration at the apex.

The Newton-Rhaphson iteration routine was used to find the maximum forward velocity at the corner apex, which corresponded to a 16m minimum path radius. This was found to be 15 m/s with a steer angle of 6.6° in all circumstances and corresponds to a lateral acceleration of 13.4 m/s.

MANOEUVRE 1 (NO BRAKING) - Figure 3 shows graph of the lateral acceleration against time for each technique, during the cornering phase. As expected the steady state technique shows an unrealistic approximation of the vehicle's performance, due to the fact that the vehicle is instantaneously at its peak lateral acceleration and minimum path radius. The quasi-static and transient simulation gently builds up the lateral acceleration as the steer angle is increased to its optimum value, which is more realistic. The transient response however, also shows the dynamics of the yaw response as the vehicle's lateral acceleration and path radius change and take time to reach their final value. This gives a more realistic simulation of how the actual vehicle performs.

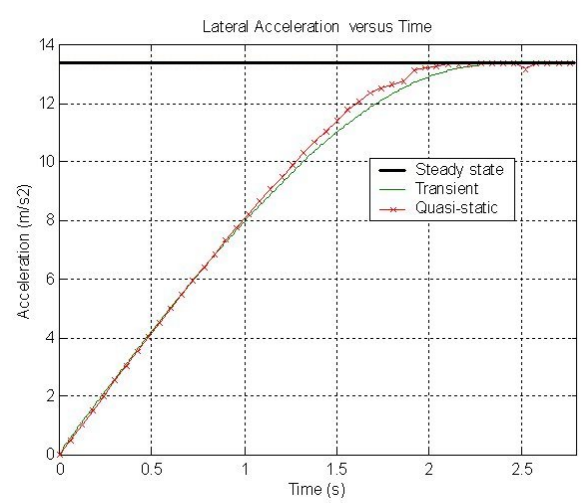

Figure 3 – Manoeuvre 1: Graph of lateral acceleration versus time for all 3 techniques.

MANOEUVRE 2 (BRAKING) - Figures 4, 5 and 6 shows graphs of the lateral versus longitudinal acceleration, forward velocity time and vehicle position respectively for each technique, during the cornering phase. Again the steady state technique is unrealistic as there is no longitudinal acceleration during cornering and it corners at constant speed. The quasi-static and transient techniques again show similar results as the vehicle speed is decreased towards the corner apex and the lateral acceleration is built up, as the longitudinal acceleration is diminished, and reaches a maximum near the apex. The transient solution again also shows the extra effects of the vehicle's yaw response as the simulation values reach their final values and follows the edge of the 'friction circle' [1] much more closely.

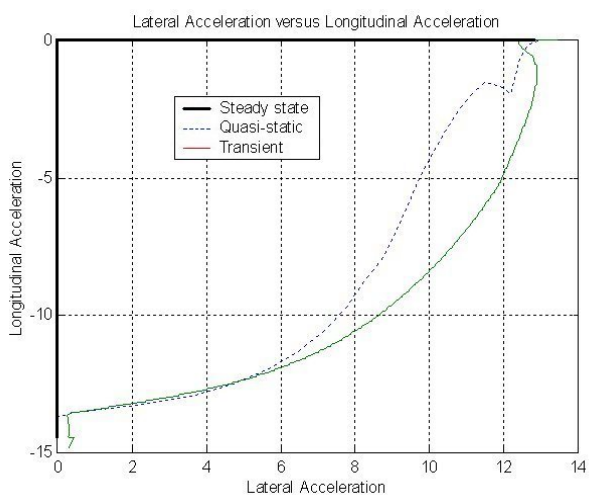

Figure 4 – Manoeuvre 2: Graph of longitudinal versus lateral acceleration for all three techniques.

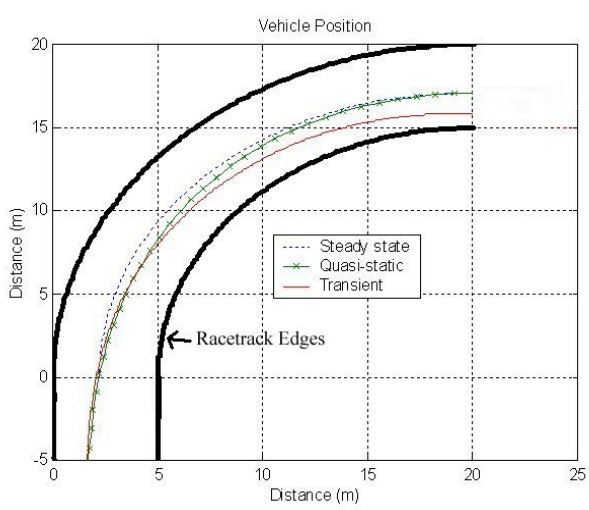

Figure 5 – Manoeuvre 2: Graph of vehicle position for all 3 techniques.

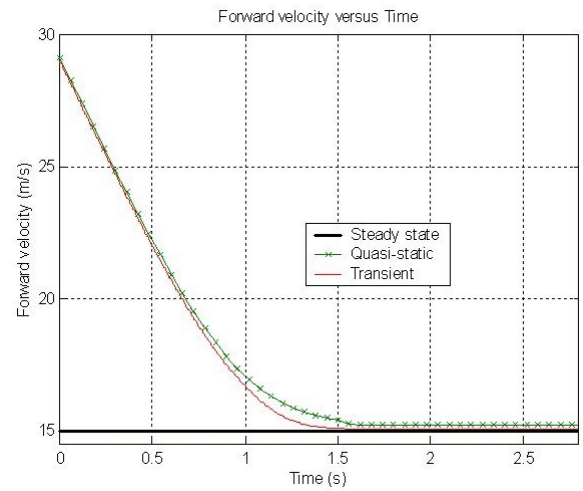

Figure 6 – Manoeuvre 2: Graph of forward velocity versus time for all 3 techniques

The main aim of lap time simulation packages is to give comparative lap times so that the vehicle's parameters can be optimised so that it will complete that lap in the shortest time. Therefore the overall time each technique simulates the vehicle completing the manoeuvre is important, as this will effect any comparison study. This only applies to the second j-turn manoeuvre where braking is involved because the first j-turn is simulated at constant speed and all three modelling techniques show the vehicle completing the manoeuvre in the same time. Table 1 shows a comparison between the three different techniques in the second manoeuvre.

Technique	**1. Steady state (time, vel.) (secs, m/s)**		**2. Quasi-steady state (time, vel.) (secs, m/s)**		**3. Transient (time, vel.) (secs, m/s)**	
Section One: Straight line Braking	1.07	30 ➔ 15	0.45	30 ➔ 23.3	0.45	30 ➔ 23.3
Section Two: Cornering and Braking	1.78	15	2.38	23.3 ➔ 15	2.36	23.3 ➔ 15
Total:	2.85	-	2.73	-	2.71	-

Table 1 – Manoeuvre two: performance comparison study.

DISCUSSION

In all three modelling strategies the vehicle reached the apex in approximately the same time and followed similar paths. This shows there is a good correlation between the techniques and that there would not be much difference in overall lap times in each case. However, in the second manoeuvre, the quasi-static and transient have produced different paths and each may require different driver inputs to follow the same path.

Unfortunately no data logger data is available and a detailed comparison study with each of the different solution techniques with the actual vehicle's performance is not possible. However the overall performance of the actual vehicle has been measured under steady state accelerations. Table 2 shows a good correlation between what has been observed during vehicle testing and simulated results.

Value	Measured Value	Simulated Value
Maximum Lateral Acc. on constant radius steer pad test (10m path radius)	12.8 m/s^2	13.4 m/s^2
Steer Wheel Angle at Maximum Lateral Acc. (10m path radius)	Approx. 60°	65°
Average Longitudinal Acc. in full stop from 40 ms^{-1}	14.7 m/s^2	14.9 m/s^2

Table 2 – Measured and simulated overall performance values for 2000 Leeds University Formula SAE car.

Examining figures 3 to 6 more closely demonstrates the differences in the various strategies. The major inaccuracy is demonstrated by the steady state strategy due to the fact that the vehicle is simulated undergoing longitudinal and steady state lateral acceleration separately, which does not occur in real life and produces a longer time to complete the manoeuvre. The quasi-static and transient solutions demonstrate how the driver uses combined accelerations to complete the corner. Both techniques show a similar response when the same inputs are used but the transient model includes the yaw response of the vehicle, see Figure 7.

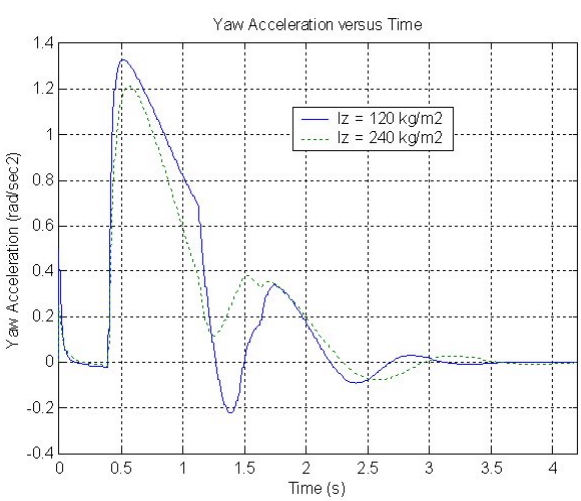

Figure 7 – Yaw acceleration response of vehicle using transient strategy in combined braking and cornering manoeuvre.

Figure 7 also shows the effect on vehicle response of doubling the vehicle's yaw inertia from 120 kg/m^2 (its actual value) to 240 kg/m^2. It can be seen that not only is the yaw acceleration lower for the larger inertia value, but also the vehicle takes longer to settle down to its steady state values. As well as this the vehicle takes longer to respond to the driver's inputs, as expected. A responsive racing car is therefore one with a low yaw inertia value.

It can also be seen from figure 7 that the transient solution technique takes into account the time the vehicle takes to respond to the driver inputs (as the simulation does not wait for the simulation to reach its steady state value at each time point). Thus the effect of a greater number of vehicle parameters on the overall lap time, in this case yaw response of the vehicle, could be explored. If this solution technique is used in a lap time simulation it would allow the racecar engineer to minimise lap times by tuning other vehicle parameters which effect the yaw response of the vehicle, including wheelbase and inertia.

The model could also be extended to include other dynamic effects. These include the effect of the suspension dampers on the response of the sprung mass in pitch, roll and bump and the effect of tyre lag on the vehicle's yaw response.

CONCLUSION

The paper has shown how three different modelling techniques have been used to explore the effects of different vehicle dynamic solution techniques in relation to racing car lap time simulation. From the results it can be seen that the difference in overall time would not be great between the different solution techniques.

However the transient solution, although more complicated, takes into account vehicle factors that are not accounted for in the other solution techniques. A fully transient solution to lap time simulation could thus be sought which would allow more accurate tuning of a greater number of vehicle parameters. This would allow the race engineer to minimise the actual vehicle's lap times.

A full lap transient solution will be the subject of future studies.

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