# Shock Absorber Mathematical Modeling 

Luciano C. Andreotti
Cofap-Cia. Fabricadora de Peças
Sérgio N. Vannucci
Cofap-Cia. Fabricadora de Peças

## Imprimir Este Paper



VII International Mobility Technology
Conference \& Exhibit
Sao Paulo, Brazil
November 9 to 11, 1998

The appearance of the ISSN code at the bottom of this page indicates SAE's consent that copies of the paper may be made for personal or internal use of specific clients. This consent is given on the condition however, that the copier pay a $\$ 7.00$ per article copy fee through the Copyright Clearance Center, Inc. Operations Center, 222 Rosewood Drive, Danvers, MA 01923 for copying beyond that permitted by Sections 107 or 108 of U.S. Copyright Law. This consent does not extend to other kinds of copying such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

SAE routinely stocks printed papers for a period of three years following date of publication. Direct your orders to SAE Customer Sales and Satisfaction Department.

Quantity reprint rates can be obtained from the Customer Sales and Satisfaction Department.

To request permission to reprint a technical paper or permission to use copyrighted SAE publications in other works, contact the SAE Publications Group.


All SAE papers, standards, and selected books are abstracted and indexed in the Global Mobility Database.

No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

## ISSN 0148-7191

## Copyright 1998 Society of Automotive Engineers, Inc.

Positions and opinions advanced in this paper are those of the author(s) and not necessarily those of SAE. The author is solely responsible for the content of the paper. A process is available by which discussions will be printed with the paper if it is published in SAE Transactions. For permission to publish this paper in full or in part, contact the SAE Publications Group.

Persons wishing to submit papers to be considered for presentation or publication through SAE should send the manuscript or a 300 word abstract of a proposed manuscript to: Secretary, Engineering Meetings Board, SAE.

# Shock Absorber Mathematical Modeling 

Luciano C. Andreotti<br>Sérgio N. Vannucci<br>Cofap Cia. Fabricadora de Peças

Copyright © 1998 Society of Automotive Engineers, Inc


#### Abstract

The objective of this work is to provide the engineering department with a simple and useful tool to predict the valve pieces to be used in a shock absorber during the design phase, in order to diminish the experimental work to be done during the vehicle ride test and prototype manufacturing. This tool is a group of equations that can predict the force behavior of the shock absorber and the corresponding valves according to the preliminary shock absorber force x velocity curve defined by the vehicle suspension engineers.


## INTRODUCTION

This project about Shock Absorber Mathematical Modeling is concerned with equations that lead us to know shock absorber force related only to valve code, through measurements and experiments. Such model will support new valve codes creation. At first, we worked with $35-\mathrm{mm}$ diameter piston but it is possible to extend that model for others, for example with P.T.F.E. piston ring.
We tried to isolate and to simplify the shock absorber real effects through boundary conditions in order to measure each effect separately and to compare real with theoretical behavior. The shock absorber with traditional design consists in three sets of valves. One group is in the piston assembly and another similar in concept in the base valve. The piston valves control mainly the rebound phase and the base valve the compression phase. (Fig.1).
As the bleeds, springs and blow-off present different energy dissipation, each one must be modeled separately and total effect in the shock absorbers shall be obtained by the "blending " of the curves in the transition of the valve operation.


Fig. 1 - Shock absorber components

## BLEEDS VALVES APPROACH

We start with bleed valves measurements because that's the first effect when shock absorber is running at low velocity (below $100 \mathrm{~mm} / \mathrm{s}$ theoretically). Before defining the experiments we must understand bleed valve features. Broadly speaking, bleeds are small passages $\left(10^{-2} \mathrm{~mm}^{2}\right)$ that control piston oil flow. The first problem was how to neutralize other variables that interact in shock absorber behavior. We changed the shock absorber structure blocking the disc movements (replacing the spring by a washer). That block would lead us to a stringent condition if we increase the velocity. Experimental calculations provided a maximum force value ( 9000 N ), so we extended our tests to this critical value. Let us now summarize the procedure to the measurements. By the valves code table, we selected a piston and then choosen eleven bleeds samples which provide equations shown in table 1, where bleeds code is an item related to passage area, v is a velocity in $\mathrm{mm} / \mathrm{s}$ and $\mathrm{R}^{2}$ is a error factor that is better as it approaches to one ( 1 is perfect). After we tested the shocks and were obtained the following curves, where the thin line is the model and the thuck is the experimental result.


Graphic 1. Bleeds $8,0^{*}$ in action


Graphic 2. Bleeds $17,0^{*}$ in action

## SPRINGS APPROACH

It's a hard task to isolate this component because its features are related to a large number of variables (composition, thermal treatment, diameters, etc), so we adopted a high quality spring ideal model. To complete boundary conditions we used the higher area passage piston and considered the useful velocity band $100-900 \mathrm{~mm} / \mathrm{s}$ where influence of spring is maximum. Again, using the valves code table, we constructed table 2 with spring samples.


Graphic 3. Spring 35 lbs . in action


Graphic 4. Spring 100 lbs . in action

## PISTON APPROACH

In discussing the shock absorber at high velocity, we are concerned with pistons, which have the greatest influence. This time all pistons available were used for calculations. We supposed the spring was totally compressed leading to a high velocity situation. The results in table 3 were obtained with $300,900,1050$ and a $1200-\mathrm{cpm}$ frequency with $50-$ mm stroke, where the main influences are the passages in the piston space reserved for blow-off.


Graphic 5. Piston R in action

## ABACUS CONSTRUCTION

The project purpose was to build an abacus that helped us on a shock absorber elaboration, a first step towards its behavior. We constructed the abacus composing bleeds curves (low velocity), spring curves (medium velocity) and piston curves (high velocity) in a unique graphic, to analyze a certain valve code for rebound force.


Graphic 6. Shock absorber abacus.

## RESULTS

The mathematical models can be used in two ways. When we have a proposed "valve code" we can predict the force x velocity curve using the equations developed. Another way is to take the proposed force x velocity curve given by the suspension engineers and describe it by a mathematical model for each valve operation region. With this curves we can define in advance which pieces shall be used in valve assembly in order to actually reproduce this curves in the prototype construction.
With this process we can design the shock absorber valves in less time and smaller cost in the prototype construction.

| Bleeds code | Force | $\mathbf{R}^{2}$ | Equation |
| :---: | :---: | :---: | :---: |
| $1,0^{*}$ | $1,8934 \cdot \mathrm{v}^{1,5673}$ | 0,9998 | Potential |
| $1,5^{*}$ | $0,5830 \cdot \mathrm{v}^{1,8131}$ | 0,9998 | Potential |
| $2,5^{*}$ | $0,5341 . \mathrm{v}^{1,7255}$ | 0,9966 | Potential |
| $4,0^{*}$ | $0,1980 \cdot \mathrm{v}^{1,8238}$ | 0,9988 | Potential |
| $5,0^{*}$ | $0,0922 . \mathrm{v}^{1,9455}$ | 0,9972 | Potential |
| $6,0^{*}$ | $0,0038 . \mathrm{v}^{12,4403}$ | 0,9960 | Potential |
| $8,0^{*}$ | $0,0270 . \mathrm{v}^{2,083}$ | 0,9907 | Potential |
| $10,0^{*}$ | $0,0320 . \mathrm{v}^{1,9101}$ | 0,9999 | Potential |
| $12,0^{*}$ | $0,0113 \cdot \mathrm{v}^{2,0201}$ | 0,9998 | Potential |
| $14,0^{*}$ | $0,0084 \cdot \mathrm{v}^{2,0527}$ | 0,9984 | Potential |
| $17,0^{*}$ | $0,0083 \cdot \mathrm{v}^{1,9878}$ | 0,9992 | Potential |

Table 1. Bleeds equations.

| Springs <br> denomination | Force | $\mathbf{R}^{2}$ | Equation |
| :---: | :---: | :---: | :---: |
| AM 8-8 | 286,84.Ln(v) $-1223,3$ | 0,9019 | Logarithmic |
| AM 15-8 | $268,63 \cdot \operatorname{Ln}(\mathrm{v})-899,47$ | 0,9093 | Logarithmic |
| AM 20-8 | $315,35 \cdot \operatorname{Ln}(\mathrm{v})-1060,7$ | 0,9227 | Logarithmic |
| AM 35-8 | $446,64 \cdot \operatorname{Ln}(\mathrm{v})-1490,6$ | 0,9988 | Logarithmic |
| AM 45-8 | $452,51 \cdot \operatorname{Ln}(\mathrm{v})-1228,1$ | 0,9972 | Logarithmic |
| AM 60-8 | $583,39 \cdot \operatorname{Ln}(\mathrm{v})-1724,4$ | 0,9960 | Logarithmic |
| AM 75-8 | $719,10 \cdot \operatorname{Ln}(\mathrm{v})-2244,3$ | 0,9907 | Logarithmic |
| AM 100-8 | $710,97 \cdot \operatorname{Ln}(\mathrm{v})-1427,5$ | 0,9999 | Logarithmic |

Table 2. Springs Equations.

| Piston <br> denomination | Force | $\mathbf{R}^{2}$ | Equation |
| :---: | :---: | :---: | :---: |
| N | $0,0005 \cdot \mathrm{v}^{2}-0,211 \cdot \mathrm{v}-5,9471$ | 0,9993 | Polynomial |
| R | $0,0011 \cdot \mathrm{v}^{2}-0,6016 \cdot \mathrm{v}+128,7$ | 0,9990 | Polynomial |
| F | $0,001 \cdot \mathrm{v}^{2}-0,472 \cdot \mathrm{v}+64,859$ | 0,9995 | Polynomial |
| T | $0,0008 \cdot \mathrm{v}^{2}-0,312 \cdot \mathrm{v}+26,734$ | 0,9987 | Polynomial |
| RA | $0,003 \cdot \mathrm{v}^{2}-1,2658 . \mathrm{v}+268,85$ | 0,9942 | Polynomial |
| BR | $0,0059 \cdot \mathrm{v}^{2}-3,883 . \mathrm{v}+785,86$ | 0,9983 | Polynomial |
| TR | $0,0046 \cdot \mathrm{v}^{2}-0,587 . \mathrm{v}-126,65$ | 0,9999 | Polynomial |
| TH | $0,001 \cdot \mathrm{v}^{2}-0,446 \cdot \mathrm{v}+21,434$ | 0,9984 | Polynomial |

Table 3. Piston Equations.

